**A New Method for Constructing Penrose-like Tilings by Using Traditional Iranian Patterns**

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In 1974, R. Penrose, a prominent British mathematician and physicist, introduced a new kind of tiling ([3]) whose characteristics were different from the known tilings. They had "*global fivefold symmetry",* which means that there is a point that if we rotate the tiling in multiples of $\frac{2\pi}{5}$ around the tiling fits with the original one, and they are non-periodic, i.e., there is no vector that if we transform the tiling by that vector, the new tiling fit with the original one. A few years later, physicists discovered some structures which are called "*quasicrystal"* and they found that a relevant mathematical model for describing (some of) these structures are Penrose tilings (see [7]). In recent years many scholars have suggested similarities and (possible) relations between Penrose tilings and some traditional Iranian-Islamic tilings which are based on the regular decagon. Perhaps the most famous article on this subject is written by P. Lu and P. Steinhart ([4]), which even attracted many social media around the world. In this article a set of five tiles (which, following [4], we call them "*Gireh"* tiles) were introduced and it is shown that those tiles have a self-similarity like property, i.e., we can construct a similar tile with a larger scale using some arrangements of tiles with smaller scales, a characteristic that also holds for Penrose tiles.

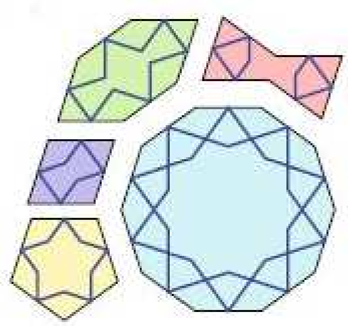


Figure3: Gireh tiles

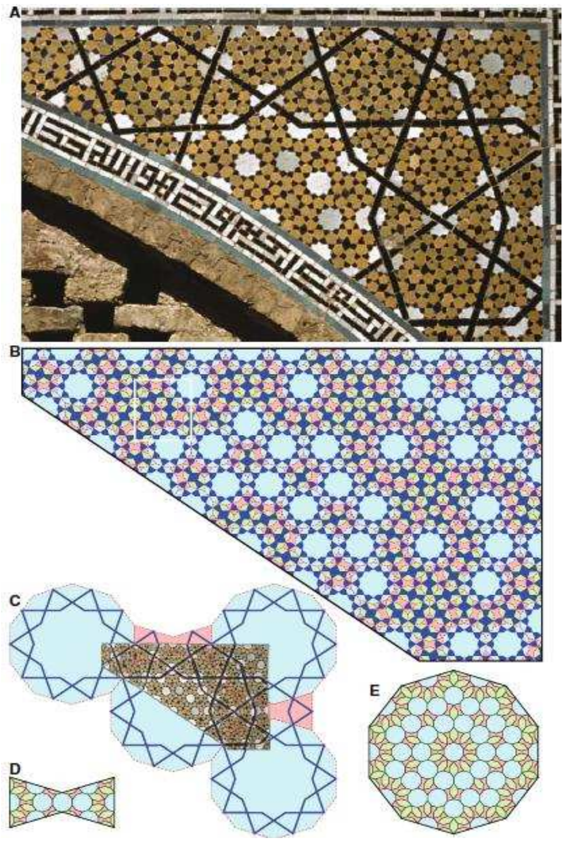



Figure4: Generating Darb-e-Imam pattern using self-similar divisions of Gireh tiles (from [4])

Lu & Steinhart’s main example is a tiling in Darb-e-Imam shrine (15th century, Isfahan, Iran) (Figure 4). They claimed that *“…and by the 15th century, the tessellation approach [using Gireh tiles] was combined with self-similar transformations to construct nearly perfect quasi-crystalline Penrose patterns, five centuries before their discovery in the West” ([4]).* Their claim has been criticized by some authors (for example [1]), but, whether all their claims are justified or not, it is clear that there are some relations between these two types of tilings and we can construct tilings with characteristics like Penrose tilings by using traditional Iranian-Islamic patterns (here after, for simplicity, we use the term “traditional patterns” instead of “traditional Iranian-Islamic patterns”).

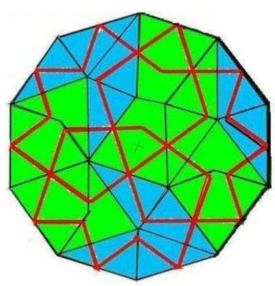
In this article we introduce a new method for constructing Penrose-like tiling using some traditional motives. These motives are:

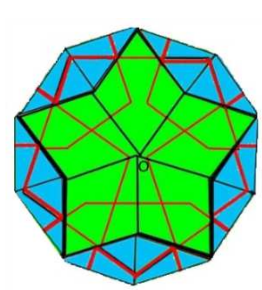
They are used in a type of pattern called by traditional master, “Kond-e-Dah.” Here is an example of Kond-e-Dah patterns:

(Jame’ Mosque, Isfahan)

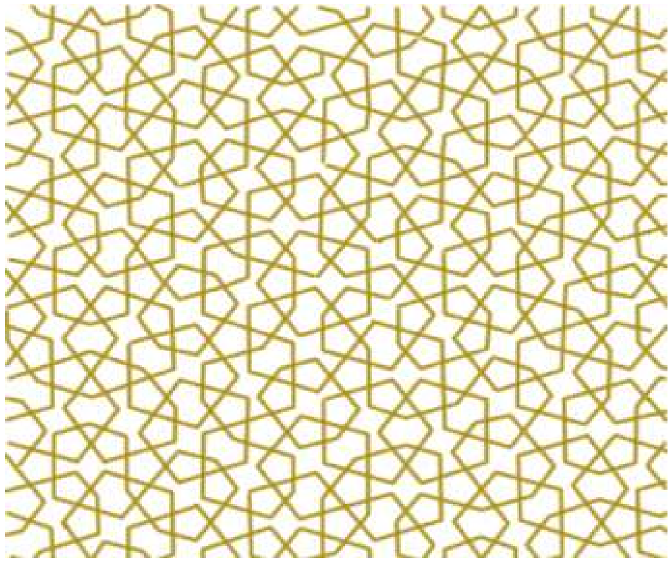
We prove that they are non-periodic and have global fivefold symmetry with exception of finitely many tiles. Our main theorem is this:

Theorem: A Penrose tiling can be covered by these two regular decagons if a certain type of overlap is allowed:





This is part of the resulting tiling:



**References**

[1]-P. Cromwell, *The Search for Quasi-Periodicity in Islamic 5-fold Ornament,* the mathematical intelligencer, January 2009, vol. 31, issue 1, pages 36-56.

[2]- M. H. Eslampanah, *Gerehsazi dar tazinat e honari,* [inFarsi],Farhang e iranzamin, no. 29, pages 497-561, 1375(1996).

[3]- B. Grunbaum, G. C. Shefard, *Tilings and patterns*, W. H. Freeman 1990.

[4]- P. Lu, P. Steinhart, *Decagonal and Quasi-crystalline Tilings in Medieval Islamic Architecture*, Science 315 (2007): 1106-1110.

[5]- R. Penrose*, The role of aesthetics in pure and applied mathematical research*, Bulletin of the Institute of Mathematics and its Applications **10**: 266 ff., 1974.

[6]- J. Rigby, *Creating Penrose-type Islamic interlacing patterns*, Proc. Bridges: Mathematical Connections in Art, Music and Science (London, 2006), eds. R. Sarhangi and J. Sharp, 2006, pp. 41-48.

[7]- M. Senechal, *What is a Quasicrystal*? Notices of the American mathematics society, vol. 53, number 8, 2006.