Dancing mathematical processes: stochastic and flow of motion dance

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We intend to explore, from a theoretical as well as an experimental way one facet of the relationship between mathematics and art. More precisely, the interplay of mathematics and dance, when we dance *mathematical processes* (instead of dancing mathematical proofs, as in the related work of Milner et al.). We engage then in *stochastic dance* and *flow of motion dance*.

Stochastic dance is akin to stochastic music, which was developed last century by Xenakis, but has older avatars, like Mozart, who showed how to compose minuets by tossing dice, in a similar way that contemporary choreographer Cunningham took apart the structural elements of what was considered to be a cohesive choreographic work (including movement, sound, light, set and costume) and reconstructed them in random ways.

It is well known that randomness generates shapes and forms: tossing a coin one hundred times generates a fractal-like shape, resemblant of a mountain ridge. Our main idea is that randomness also generates movements, dance and even choreographies.
To try to embody this idea amounts to exploring an enactive and experiential analogue of stochastic music, in the realm of dance, where the poetry of a choreographic spatial/floor pattern is elicited by randomness, in the form of a mathematical stochastic process, e.g., a random walk – a stochastic dance of sorts.

Among many possible random walks, we consider two simple but paradigmatic examples, embodied in the following scenarios, proposed to the students/dancers:
- a frog, jumping randomly on a row of stones, choosing right or left as if tossing a coin,
- a person walking randomly on a square grid, starting a given node, and choosing each time randomly, equally likely N, S, E or W, and walking non-stop along the corresponding edge, up to the next node, and so on.

When the dancers encounter these situations, quite natural questions arise for the choreographer, like: Where will the walker/dancer be after a while?
Several ideas for choreography emerge, which are more complex than just having one or more dancers perform the random walk, and which surprisingly change our random process into a deterministic one!

- For instance, for the first random walk, 16 dancers start at the same node of a discrete line on the stage, and execute, each one, a different path of the 16 possible 4 – jump paths the frog can follow. They would need to agree first on how to carry this out. Interestingly, their score may be non-hierarchical, without a Magister Ludi handing out scripts to every dancer. After arriving to their end node/position, they could try to retrace their steps, to come back all to the starting node. This would create quite a harmonious unfolding and infolding pattern, which could be repeated over time.

- Analogously for the grid random walk, where we may now have 16 dancers enacting the 16 possible 2-edge paths of the walker, or 64 dancers enacting the 64 possible 3-edge paths of the walker. Their collective movement would reveal then the unfolding of all possibilities for this (2 or 3 steps) random walk.

-The dancers could also enter the stage (the grid or some other geometric pattern to walk around), one by one, sequentially, describing different random paths, or deterministic intertwined paths, in the spirit of Beckett's Quadrat.

- Also, the dancers could choose their direction ad libitum, after some spinning, each time, on a grid-free stage, but keeping the same step length, as in statistician Pearson's model for a mosquito random flight.

We are interested in various possible spin-offs of these choreographies, which intertwine dance and mathematical cognition: For instance, when the dancers choose each one a different path, they will notice that their final distribution on the nodes is uneven (interesting shapes emerge). In this way, just by moving, choreographer and dancers can find a quantitative answer to the impossible question: where will the walker/dancer be after a while? Indeed, the percentage of dancers ending up at each node gives the probability of the random walker landing there.

By *flow of motion dance,* we understand the bodily enactment of a mathematical flow in a finite universe (as stage), a finite cyclic universe in our case, mathematically modelled by the integers modulo m (for a fixed natural integer m), visualized as a regular polygon with m sides.

We consider the dynamic systems defined by multiplication by a fixed integer k modulo m. Each integer modulo m is the starting point of a trajectory (its fate, or destiny, metaphorically speaking) consisting of its successive images.
See this video: https://www.youtube.com/watch?v=KRFTIEAs75M for an animation in the case m = 16 and 17 and all possible k's.

As in a flow in a river where each drop of water follows a definite trajectory, here every dancer starting at any given vertex of the polygon will follow a definite trajectory (enacting his/her destiny), which "goes forever" or ends up "falling into a sink".
This should create the visual effect of a (human) flow. Interestingly, for non-prime m (like *16)* we find attractors which can be sinks or cycles or various lengths. For $k = 8,$ we see a flow that inexorably dies out: all the dancers finally "fall" into the sink 0. For prime m (like *17),* for k = 4, the global flow (on non-zero vertices) *decomposes* in 4 disjoint flows, which are 4-cycles. A pattern which is revealed through the dancers' movements (who could also be dressed in the corresponding orbit colour).

The theoretical main thrust of our paper is that mathematical objects or processes, deterministic or stochastic, naturally generate shapes, movement and dance, through embodiment. An open question is how, besides enacting spatial patterns in the floor, could the dancers enact the flow of motion in their own bodies.

References

Milner, S.J., Duque, C. A., Gerofsky, S. (2019). Dancing Euclidean Proofs: Experiments and Observations in Embodied Mathematics Learning and Choreography. Proc. 2019 Bridges Conference (pp. 239-246)