Embodiment and competencies: A discussion of the KOM-framework of mathematical competences.

Oliver Kauffmann & Uffe Thomas Jankvist, University of Copenhagen, Denmark

Today competency has become a key construct in the educational paradigm within various domains (e.g., Sadler, 2013; Stacey, 2010; Stacey & Turner, 2014), overshadowing and replacing former, dominant constructs such as knowledge and skill. Mathematics is no exception to this. The notion of mathematical competencies, as opposed to mathematical skills and knowledge, has gained momentum within the past decades in mathematics programmes, not only in Scandinavia and Northern Europe, but also for example in Columbia and not least in the international assessments PISA (OECD, 2019). Yet, in the last three decades, we have also been witnessing a turn in cognitive sciences, social sciences, and the humanities: the turn toward understanding learning, feeling and cognizing as situational embedded, embodied activities (Varela et al.. 1991; Lakoff & Johnson, 1999; Shapiro, 2019). These conceptualisations of embodied cognition have also had impact on the understanding and investigations of learning and skills within mathematics (see e.g., Lakoff & Nuñez, 2000).

In this paper, we take a closer, critical look at the role of the body in relation to mathematical competencies. We investigate to what extent a particular, influential conceptualisation of competencies, the KOM-framework (Niss & Jensen, 2002; Niss & Højgaard, 2011), would benefit from a supplement by an embodied dimension, and discuss various ways in which this could be accomplished.

The basic, original idea of KOM was to formulate the concepts of mathematical competence and competencies "with particular regard to their possible roles in the teaching and learning of mathematics" (Niss & Højgaard, 2019, p. 10). Thus, the main thought behind the KOM-framework was that the teaching of mathematics would be able to promote the students' development of these competencies and the related kinds of 'overview and judgment'.

The overall characterization of 'mathematical competence' KOM describes as "…having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role" (Niss & Højgaard, 2011, p. 49). This overarching conception of competence spans eight distinct, yet mutually related, separate competencies, often illustrated by an 8-leafed flower, the so-called KOM-flower. The competencies are divided into two groups, the first referring to "the ability to ask and answer questions in mathematics and with mathematics" (Niss & Højgaard, 2011, p. 50); the second to "the ability to deal with mathematical language and tools" (p. 50). Both of these groups have (almost exclusively) explicitness (through language) as characteristics, and generally the competencies are conceived as cognitive of nature (cf. e.g., Niss & Højgaard, 2019, p. 18).

The authors of the original KOM-report recently found reasons to revisit the conceptualization of the basic notions "in order to provide an updated version of the original conceptual framework and terminology." (Niss & Højgaard, 2019 p. 9), a paper in which they also give a concise, but instructive exposition of the relation between procedural skills and mathematical competencies (Niss & Højgaard, 2019, p. 20). Their exposition is a pertinent starting point for a discussion of the role (or no-role) of the body for mathematical competence.

They suggest that skills can be seen as necessary, but not sufficient for the competencies. They conceive the relation between skills and competencies as a specific type of compositional relation: The existence of competencies relies on the existence of procedural skills, but without the competencies being reducible to these skills. The procedural skill of performing rule-based transformations of algebraic expressions in different mathematical domains for instance, is a precondition for competency with symbol-manipulation and formalism. Furthermore, the authors conceive this compositional relation with a metaphor from chemistry: Huge molecules (e.g. polymers) composed out of atoms, where competencies are comparable with the emergent properties of such molecules, the atoms of which (with their lower-level properties) are the procedural skills.

But does this 'picture' fit with the distinction between skills and competencies? Clearly, the higher-order properties can be analysed and become the object of scrutinisation, differentiated as they are in the eight different leaves of the KOM-flower. They are all more or less amenable to cognitive methods of research. Still, since these competencies rely on the skills, in the sense that the competences do not come into existence at all without the skills, a pertinent discussion is why the skills (or a subset thereof) do not appear as part of the competencies? There might be several reasons for *not* including skills in competencies: Perhaps there are too many skills involved to list—"probably hundreds" (Niss & Højgaard, 2019, p. 20) — or perhaps procedural skills are not explicitly available, when we evaluate, grade or otherwise probe students' performances? In addition, procedural skills are learned differently compared to explicit contents. Yet, these reasons are just pragmatic reasons and do not raise an objection in principle against including (some) procedural skills in the set of competencies. Indeed, the authors of the KOM-framework, in their very reconceptualization of mathematical competence, rephrase this as a skill themselves: "Mathematical competence is someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations." (Niss & Højgaard, 2019, p. 12) 'Readiness to act in such circumstances' appears as a (meta)skill.

The literature on embodied mathematical cognition gives examples to the effect that mathematical skills from a learning perspective ultimately rely on bodily skills (Lakoff & Nuñez, 2000; Nuñez et al., 1999), gives reasons for pursuing reconceptualisations of procedural knowledge in mathematics (e.g. Star, 2005), and gives evidence to the effect that certain mathematical tasks can be accomplished by alternative, embodied, enacted procedures (e.g. Abrahamson et al., 2021; Donovan & Alibali, 2021; Hutto et al., 2015).

On the background of such examples and the specific compositional picture of the KOM-framework, we discuss how a model of mathematical competences should benefit.

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