Rep-Tile Tangram

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Goal of the communication: connecting math, art, creativity and technology in teaching
Rep-Tiles, or repeated tilings, are tilings where each tile contains smaller copies of itself. The term “Rep-Tiles” was suggested by Solomon W. Golomb and popularized by Martin Gardner (Gardner, 1991). In classic Rep-Tiles one shape repeats itself, with a fixed scaling factor, such as the hexagon in Figure 1.

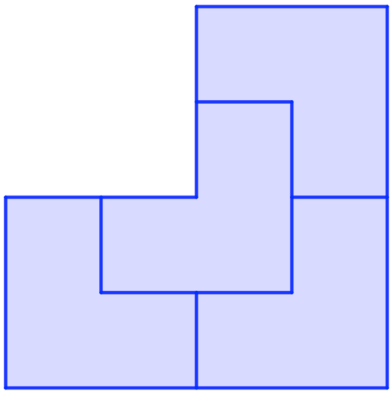

Figure 1: a rep-tile hexagon, composed from 4 scaled-down copies of itself.

In this contribution we will create Rep-Tiles from tangram. A tangram is a classic puzzle made from seven pieces which are sometimes also referred to as "tans" (Figure 2). Five of these seven tans are similar triangles, so that there are only three different basic shapes: an isosceles right-angle triangle in three different sizes, a parallelogram and a square.

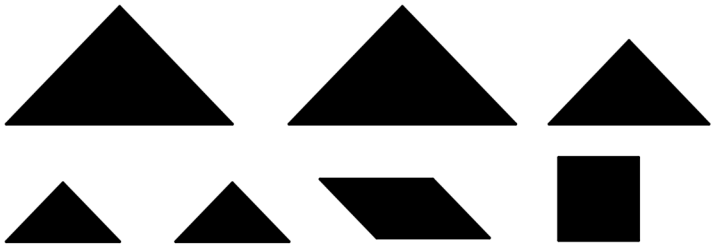

Figure 2: A full set of seven tangram pieces.

Each of the three different shapes of the tangram game can be reproduced from all pieces of the game as illustrated in Figure 3. That way, we can create repeated tilings with tangram pieces. As opposed to the original Rep-Tiles, each piece of tangram will be composed of the entire set of seven tans, and not of one shape only. There are three different scaling factors, since different

shapes cover different areas of the puzzle (see Table 1). Continuing the process of replacing tangram pieces by appropriate arrangements of sets of smaller tangram pieces, we can create self-similar tans, at least theoretically. In practice we have to stop the process of replacing shapes by ever smaller shapes at a certain point.

Figure 3:Three basic tangram shapes, the triangle, the square and the parallelogram, made from tangram shapes.

Table 1: Scaling factors of the various tans

Shape	Area a as fraction of the area of the entire puzzle	Scaling factor s_i of the shape with respect to the same shape made from all tans ($s_i = \sqrt{a}$)	Number of copies of the shape in a full tangram set
Large triangle	$^1/_4$	$^1/_2$	2
Medium triangle	$^1/_8$	$^1/_{2\sqrt{2}}$	1
Parallelogram	$^1/_8$	$^1/_{2\sqrt{2}}$	1
Square	$^1/_8$	$^1/_{2\sqrt{2}}$	1
Small triangle	$^1/_{16}$	$^1/_4$	2

The Rep-Tile tangram can be used in various ways in the mathematics classroom. The Rep-Tile tangram teaches similarity, self-similarity and can be used as an introduction to studying fractals. It is relevant for computer classes since it allows the creation of beautiful pictures by iteration. When teaching the use of spread-sheet programs, it can be an interesting task to calculate the areas of the various tangram shapes for a given scaling factor. The Rep-Tile tangram is a suitable tool for teaching the right balance between competitive and collaborative behavior, as the following activity will show.

For this classroom activity each participant will receive the contour of a tangram shape (a triangle, a square or a parallelogram) and a classic set of seven tangram pieces printed on a sheet of paper. The task is to assemble the seven tans to fill the given contour. At this point, every participant will have a tangram shape made from smaller tangram shapes (for example a square as in Figure 2).
In phase two, every participant has to join six other participants so that the group's shapes will complete each other to create a full tangram set. For example: a participant who assembled a square in phase one needs to find five participants holding triangles of appropriate sizes and an additional participant holding a parallelogram.

Each team of seven participants needs to assemble a tangram shape (a triangle, a square or a parallelogram). The result will be a tangram shape made from tangram shapes that are made from tangram shapes. The winner will be the team that solves the task first.

In summary, various teaching activities, such as solving puzzles, calculating scaling factors, creating fractals or even programming tasks can be based on the Rep-Tile tangram, which is both interesting due to its mathematical properties, and beautiful.

References

Gardner, M. (1991). Rep-Tiles, Replicating Figures on the Plane. In M. Gardner, *The Unexpected Hanging and Other Mathematical Diversions* (pp. 222-233). Chicago: Chicago University Press.