Characterising problem handling in the intersection between computational thinking and mathematics

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Introduction

This study is embedded in a larger project exploring the possible synergies between programming and computational thinking (CT) and mathematical digital competencies. In this intersection, problem handling or solving has a prominent place, though its meaning may not be the same. In the growing research on CT in mathematics education, it is often argued that there are strong synergies between CT and problem solving (Kallia et al. 2021; Weintrop et al., 2016). Niss and Højgaard (2019) however emphasise the importance that problem tackling in mathematics education is targeted toward *mathematical* problems and that the use of mathematics to solve extra-mathematical problems exclusively belongs to the modelling competency. This communication reports on interventions in collaboration with a mathematics teacher aimed at developing and implementing teaching materials that introduce students to problem-solving tasks that integrate CT and mathematics. Drawing on classroom observations, we seek to characterise the notion of problem handling via CT in the mathematics classroom.

Background

There is a vast tradition of problem solving (PS) in mathematics education. This tradition tends to use problem solving as a means for teaching mathematics (Cai, 2010), and thus the PS competences are not separated from the domain-specific objectives in the curriculum (Artigue & Blomhøj, 2013). In the Danish framework for mathematical competencies (KOM), Niss and Højgaard (2011) delineate problem handling as the ability to pose and solve different kinds of mathematical problems. In their recent work, they highlight that PS concerns mathematical problems, and tackling extra-mathematical problems belongs to the modelling competency (Niss & Højgaard, 2019). Furthermore, Geraniou and Janvkist (2019) contributed a theoretical networking of mathematical and digital competence (MDC), in which PS is also mentioned. One of the three main components of MDCs involves “being able to use digital technology reflectively in problem solving and when learning mathematics” (p. 43).

Wing (2006) emphasises computational thinking as “a way humans solve problems” (p. 35), and a later comprehensive definition states CT as “the conceptual foundation required to solve problems effectively and efficiently (i.e., algorithmically, with or without the assistance of computers) with solutions that are reusable in different contexts” (Shute et al., 2017, p. 142). When embedded into mathematics classrooms, the literature validates problem solving as an ultimate purpose (Kallia et al., 2021) or as a core set of practices (Weintrop, 2016; Pérez, 2018) to go about it. What all of these frameworks also agree on is that CT should enable the transfer of solution strategies to other fields.

Our work is conducted in Denmark, where CT was included in the compulsory school curriculum as an experimental subject. Two preliminary results are important to mention. First, problem handling is not invoked in any of the available resources that integrate programming and CT into mathematics teaching (Elicer & Tamborg, submitted). Second, the mathematics teacher with whom we collaborated strongly emphasized the necessity for students to own (delineate and pose) the computational problem (Elicer et al., 2022). After our experience implementing tasks with her students, we aim to address the following research question: How can we characterise the notion of problem tackling in the context of programming and CT in the mathematics classroom?

Empirical basis

The following discussion is based on selected episodes from a classroom experience with a 6th-grade class of 18 students. The task at hand was co-designed with the teacher over the course of several months. Students were asked to program different regular polygons in Scratch and explore the relation between the number of sides and the corresponding turning angles (cf. Elicer et al., 2022). Students were to use such figures to draw a city skyline of their choice. Data were collected in the form of video and audio recordings of the three 90-minute sessions and a post-intervention interview with the teacher.

Characterising problem handling

Our communication presents three issues that characterise problem handling when integrating CT into mathematics education.

The first issue concerns the disciplinary nature of the problems at hand, which has several components. Figuring out the pattern or a general expression of the internal or external angle of any given regular polygon is a *mathematical* problem in the sense of Niss and Højgaard (2019), but its CT-mediated solution has a pragmatic instead of epistemic value (Geraniou & Jankvist, 2019). Instructing the Scratch pen environment to draw a generic regular polygon or an elaborated skyline is a *computational* problem and necessitates the teaching of mathematical concepts to be solved. Furthermore, drawing a skyline is closer to a *graphic design* problem, to be solved by computational means.

The second issue regards ownership; whose problem is it? Niss and Højgaard (2011) referred to the meaning of a problem being relative to the person facing it. Since the task's first iterations, the teacher insisted on framing the problem so that students put their motivation and interests in the problem to solve, as opposed to us giving them a well-delineated sequence of problems (Elicer et al., 2022). The students' ability to pose problems is then exercised by allowing them to make choices in the graphical characteristics and sequence of polygons, as well as the skyline. In that sense, the role of CT as a set of ways of thinking when handling problems (Kallia et al., 2021) should also be carefully distinguished. In the geometry task, abstraction, decomposition and modelling are more related to delineating a problem, while pattern recognition, debugging and evaluation are rather connected to its solution.

Third, CT is meant to provide solution strategies transferable to another person, machine and even discipline (Kallia et al., 2021). Proponents of CT highlight its potential to solve problems in different contexts (Wing, 2006; Weintrop et al., 2016). The aim of building general problem-solving strategies is common to mathematics education, and there is a risk of mathematics losing that territory. Students find a greater epistemic value in finding (external) angle patterns with Scratch rather than in GeoGebra (internal). The meaning of an angle in a CT environment is the preferred provider of a solution strategy over Euclidean geometry.

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