ASYMMETRY AND MATHEMATICS FOR TEACHING IN THE DIGITAL ERA

Proposal for a **remote** paper presentation at MACAS Symposium 2025

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This presentation will stem from the proposer's experience teaching technology-enhanced mathematics education courses to K-12 teacher candidates. It will focus on the curricula of the courses through the lens of asymmetry as a "fall for parity" (Gardner, 1964). The intent of such a focus is to demonstrate what both explicit and hidden presence of asymmetry in the courses brings to the study of mathematics for teaching (Cuoco, 2001). This study will be evaluated in terms of the interplay between education and mathematics (Wittmann, 2021).

At the early elementary level, young children can be given symmetrical shapes (e.g., isosceles triangles and trapezoids, rectangles, rhombuses) to put them together and see whether the combination of two (or more) symmetrical figures (with at least one side length being equal among the shapes) retains the attribute of symmetry or not (i.e., becomes asymmetrical). They can trace a symmetrical figure on a piece of paper and then use paper folding, holes punching, or scissors to demonstrate the relationship between the two parts. At the upper elementary level, using the context of pizza sharing one can create an environment to deal with fractions visually without using any mathematics other than counting and measuring. In such an environment, dividing fairly m identical circular pizzas among n people, $n > m$, with the minimal number of slices using a pizza wheel can result in either mutually asymmetrical or symmetrical slices. Visualization allows one to see the results of division of pizzas through the lens of symmetry and asymmetry.

Likewise, high school algebra provides teachers with opportunities to talk about asymmetry in the context of digital fabrication of the images of loci of inequalities with parameter. Connecting algebra to geometry allows one to learn how visual is controlled by symbolic and vice versa, how certain changes in geometric shapes create conditions for analytic description of their boundaries, and to appreciate the reciprocity of symmetry with asymmetry through the sensitivity of analytics. This knowledge connects mathematics and mathematics education and is applicable to teaching the subject matter as "a creative art because mathematicians create beautiful new concepts" (Halmos, 1968, p. 388).

Many critical issues in STEM education deal with engineering problems that require knowledge of roots of one-variable polynomials and their location in the coordinate plane (Leipholz, 1987). Investigating quadratic equations with parameters in the technological paradigm enriches secondary mathematics teacher education courses with mathematical modeling explorations redolent of real research experience in STEM fields that teacher candidates need to know about as they plan to teach secondary school mathematics. An example about the location of real roots of the trinomial $x^2 + bx + c$ about an interval on the number line can demonstrate that in the (c, b)-plane of parameters to the right of the vertical axis, a symmetrical outcome when the interval includes the smaller root only is very small in comparison with asymmetrical outcomes when the interval does not include the roots. One can observe bifurcation of symmetry into asymmetry (and vice versa) when the parabola $y = x^2 + bx + c$ is symmetrical about the y-axis.

The entries of the classic Pascal's triangle are symmetrical within each row. The top right-bottom left diagonals of the triangle consist, respectively, of ones, natural numbers, triangular numbers, triangular pyramidal numbers, pentatope numbers, and so on. An asymmetrical rearrangement of those numbers when the diagonal with ones forms the first column, the diagonal with natural numbers forms the second column shifted down about the first one by two rows, the diagonal with triangular numbers becomes the third column shifted down about the second one by two rows, the diagonal with triangular pyramidal numbers forms the fourth column shifted down about the third one by two rows, and so on, is consequential, both educationally and mathematically. The numbers appearing in each row of the rearranged Pascal's triangle can serve as coefficients of (one-variable) Fibonacci-like polynomials (Abramovich & Leonov, 2019) having all their roots located within the interval $(-4, 0)$. Furthermore, out of two polynomials of the same degree, one polynomial has symmetrical location of roots, and another polynomial has asymmetrical location of roots within the interval. These observations would not be possible without the use of digital technology.

The joint use of Wolfram Alpha, Maple, and a spreadsheet in the context of Fibonacci-like polynomials is critical for demonstrating asymmetry and symmetry of the roots of the polynomials and the introduction of generalized Golden Ratios as cycles in the form of strings of numbers. These findings although discussed in the context of mathematics education turned out to be new for the very discipline of mathematics. This points at the importance of using the modern-day computational tools in revealing to the learners of mathematics, including teachers of the subject matter as the major custodians of knowledge in general, the hidden presence of asymmetry inside the variety of mathematical structures. This presentation, expanding the proposer's ongoing research on the use of asymmetry in K-12 teacher preparation (Abramovich, 2025), will review the examples mentioned above as allocated time permits.

References

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