Creating Music with Math: What Educators Can Learn from This Interdisciplinary Experience?

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Mode of presentation: in person

Objective

The objective of our presentation is to discuss three methods for generating music through mathematical rules: the musical dice game (17th century, Hedges, 1978), the magic square (Grosser, 1994), and the Tonnetz (Euler, 1739), and to invite the audience to experiment with them. Drawing from these three methods of generating music, we will explore how they can contribute to a pedagogical intervention focused on students' musical creativity as a means of introducing mathematical concepts.

Abstract

There is a long-standing tradition of using mathematical tools, concepts, and models to automate processes that would otherwise rely on human intervention. e.g., Automatization of calculations in the 17th century with a machine like the Pascaline (Freiman & Robichaud, 2018). Music, too, has inherited this mathematical inventiveness aimed at automating tasks, notably through machines such as the player piano and the phonograph. One question remains, however: what is an educational potential of these mathematical-musical experiences? In our presentation we will explore one form of automation support that has particularly intrigued the musical world since ancient times which is the composition of music based on mathematical rules, as illustrated by the following three examples:

1. The musical dice game "Musikalisches Würfelspiel" invented in the 18th century, was used by Mozart to compose minuets (a 17th–18th century musical form in triple meter with a moderate tempo) randomly with the help of two dice. The process involves assembling precomposed one-bar musical fragments by Mozart (segments of music

defined by a specific number of note values—see Figure 2) in sequence selecting from several possible versions based on the roll of two dice. To play the game, you have two tables of 8 columns (figure 1). Each table corresponds to 8 bars (16 bars in total) and one “Musical Table” composed by Mozart. You need to roll a total of 16 times two dices (the numbers 1 to 12 represent the sum of the two dice). Each dice roll corresponds to 1 bar. Ex.: For the bar I, if the 2 dice roll gives a 7 (for example, 3 and 4), you must write bar 104 (the number in the box at the intersection of number 7 and bar I), which is found in the “Musical Table” (figure 2) composed by Mozart. After all 16 dice rolls are completed, the result is a 16-bar minuet, selected from trillions of possible combinations!

	I	II	III	IV	V	VI	VII	VIII
2	96	22	141	41	105	122	11	30
3	32	6	128	63	146	46	134	81
4	69	95	158	13	153	55	110	24
5	40	17	113	85	161	2	159	100
6	148	74	163	45	80	97	36	107
7	104	157	27	167	154	68	118	91
8	152	60	171	53	99	133	21	127
9	119	84	114	50	140	86	169	94
10	98	142	42	156	75	129	62	123
11	3	87	165	61	135	47	147	33
12	54	130	10	103	28	37	106	5

	I	II	III	IV	V	VI	VII	VIII
2	70	121	26	9	112	49	109	14
3	117	39	126	56	174	18	116	83
4	66	139	15	132	73	58	145	79
5	90	176	7	34	67	160	52	170
6	25	143	64	125	76	136	1	93
7	138	71	150	29	101	162	23	151
8	16	155	57	175	43	168	89	172
9	120	88	48	166	51	115	72	111
10	65	77	19	82	137	38	149	8
11	102	4	31	164	144	59	173	78
12	35	20	108	92	12	124	44	131

Figure 1 – Tables of bar numbers (Kocher, 2016, p. 2)

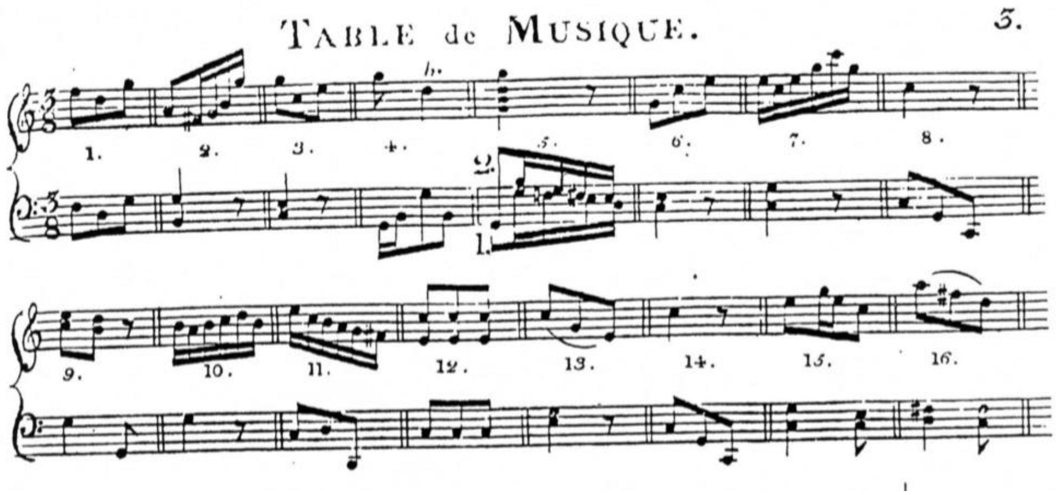

Figure 2 – excerpts of the “Musical table” (Mozart, 1793, p. 3).

2. Historically, the magic square was consisted of a n x n grid of numbers in which each row, column, and diagonal adds up to the same total. For example, in a 3×3 magic square filled in with numbers 1 to 9, all rows, columns, and diagonals sum up to 15. Other types of magic squares called *Latin Squares* were studied, among others by Euler (1782). To combine it with music, the numbers in the magic square are mapped to musical elements—such as pitch, duration, intervals, or intensity. For example, we could conceive a 3x3 Latin square where each box contains duration in eighth notes (1 beat = 2 eighth notes).

3	2	4
4	3	2
2	4	3

Figure 2 – magical square

If we add the numbers in row 1: 3 + 2 + 4 = 9 eighth notes. If we add the numbers in column 1: 3 + 4 + 2 = 9 eighth notes and if we add the numbers in diagonal: 3 + 3 + 3 (or 4 + 3 + 2) = 9 eighth notes. If we assign a rhythmic pattern to each number: 2 = two eighth notes ♪ ♪; 3 = a dotted quarter note ♩.; 4 = a quarter note + two eighth notes ♩ ♪ ♪, we get a rhythmic structure to compose with where, no matter which direction you read the table, the total remains constant. (For more complex examples with so called *Durer Squares*, see, e.g., Besada et al. (2022)).

3. The *Tonnetz*, conceived by Leonhard Euler in the 18th century and expanded in the Twentieth century by Hyer (1989) and Lewin (1987), is a graphical representation that highlights the harmonic relationships between pitches within the context of Western tonal music. It makes all pitch combinations visible, allowing for composition either randomly, through mathematical formulas or musical AI (e.g., Google Magenta). Here is a simplified *Tonnetz*:

F
↙ ↘
C ← A
↙ ↘ ↙ ↘
G ← E ← C#
↓ ↓ ↓
D ← B ← G#
↘ ↙ ↘ ↙
F# ← D#
↘ ↙
A#

Figure 3 (modified form source: Euler, 1739, p. 147)

A, B, C, D, E, F, G, A#, C#, D#, F#, G# are the musical notes. Neighbouring notes in the Tonnetz are connected by different intervals (difference in pitch between two sounds): horizontal ← = minor third (e.g., C–A); diagonal ↘ = major thirds (e.g., C–E); diagonal ↙ = perfect fifths (e.g., C–G).

One often cited quote from Leibnitz (see, for example, Archibald, 1924, p. 1) reflects hidden connections between music and mathematics: "Music is a hidden exercise in arithmetic, of a mind unconscious of dealing with numbers" supported a holistic perspective where both disciplines, seemingly ‘sharply contrasted’ are indeed ‘bound together’, "supporting one another as if they demonstrated the hidden bond which draws together all activities of our mind" (Archibald, 1924, p. 1).

Based on these historical examples, we will discuss a potential of designing teaching and learning activities from the perspective of each discipline, and, what we find the most important for MACAS community, their interdisciplinary and transdisciplinary connections from a more integrative view of education (example, helping students discovering their interests and forces).

Our previous work with Grades 7-8 teachers (one teaching mathematics and the other science) on integrative scenarios help them to make several connections to the curricula (e.g., fractions in mathematics). In the interviews, students were sharing their curiosity about connections between music and mathematics they would never think about (Robichaud & Freiman, 2022).

On the other side, we hypothesize that these connections between music and mathematics would foster students' creativity across various school disciplines. Through creativity, these activities enabled learners to approach learning in new ways - by producing novel ideas, identifying original solutions, and articulating innovative viewpoints (Runco & Jaeger, 2012) important for every citizen in our modern society locally (connection to provincial strategic plans for education in NB) and globally (e.g., OCDE (2021) and UNESCO (2023)’s policies).

Besides the above-mentioned pragmatic goals, our inquiry focuses on what is commonly referred to as *in situ creativity,* where the act of creation occurs precisely within the context in which it is intended to generate impact. In our case, this process is mediated through the exploration of the affordances of automated tools designed for music expression. We think these experiences could foster a deep sense of inspiration, belonging, and wonder. As Reid and Davidson (2018) put it reporting the results of the Working Group on in situ creativity at the CMESG (Canadian Mathematics Education Study Group) Annual Meeting: “It has sometimes been asked what 'work' a CMESG working group accomplishes. In our case we learned. That is, we changed our beings in interaction with our worlds. We left as different people than we arrived" (p.8).

This insight will guide pedagogical part of our presentation while connecting these examples to our work with teachers on fostering in situ creativity in interdisciplinary math-music contexts (Robichaud and Freiman, 2022) to extend our investigation of how the creative process and the learning of mathematics and music could be enacted when students engage with the musical dice game, the magic square, and the *Tonnetz*.

Bibliography

Archibald, R. C. (1924). Mathematicians and music. *The American Mathematical Monthly*, *31*(1), 1-25.

Besada, J. L., Guichaoua, C. & Andreatta, M. (2022). From Dürer's Magic Square to Klumpenhouwer Tesseracts: On Melencolia (2013) by Philippe Manoury. *Music Analysis*, *41*(1), 145-182.

Euler, L. (1739/2015). Écrits sur la musique, Volume 1 - Tentamen novae theoriae musicae. GREAM

Euler, L. (1782). Recherches sur une nouvelle espèce de carrés magiques. *Verhandelingen uitgegeven door het zeeuwsch Genootschap der Wetenschappen te Vlissingen*, 85-239.

Freiman, V. & Robichaud, X. (2018). A Short History of Computing Devices from Schickard to de Colmar: Emergence and Evolution of Ingenious Ideas and Technologies as Precursors of Modern Computer Technology. *Computations and Computing Devices in Mathematics Education Before the Advent of Electronic Calculators*, 339-364.

Hedges, S. A. (1978). Dice music in the eighteenth century. *Music & Letters, 59*(2), 180-187.

Hyer, B. (1989). *Tonal intuitions in "Tristan und Isolde"* [Dissertation].

Kocher, P. (2016). Rethinking 'Open Form'.
https://www.researchgate.net/publication/311570655_Rethinking_'Open_Form'

Lewin, D. (1982). A Formal Theory of Generalized Tonal Functions. *Journal of Music Theory 26*(1) 32-60.

Mozart, W.A. (1793). *Anleitung*. https://s9.imslp.org/files/imglnks/usimg/7/7c/IMSLP798374-PMLP1260062-attr_mozart_g.270.g.-2.-_Anleitung_Walzer_oder_Schleifer_mit_zwei_Wu-rfeln_zu_componiren.pdf

OCDE (2021). *Future of Education and Skills 2030*. https://www.oecd.org/education/2030-project/

Reid, D. & Davidson, N. (2018). *Mathematics in situ*. Canadian Mathematics Education Study Group. Acadia University, NS, Canada. http://www.cmesg.org/wp-content/uploads/2018/11/Report-of-Working-Group-F.pdf

Robichaud, X. & Freiman, V. (2022). Wonders of Mathematics Through Technology and Music Creativity in a School Setting. In *Mathematics and Its Connections to the Arts and Sciences (MACAS) 15 Years of Interdisciplinary Mathematics Education* (pp. 431-453). Cham: Springer International Publishing.

Runco, M. A. & Jaeger, G. J. (2012). *The standard definition of creativity*. *Creativity Research Journal*, 24(1), 92–96.

UNESCO (2023). *Reimagining our futures together: A new social contract for education*. https://unesdoc.unesco.org/ark:/48223/pf0000379707

Wills, H. (1989). Magic with magic squares. *The Arithmetic Teacher*, *36*(8), 44-49.