Dynamic geometry and automated reasoning for mathematics and arts education: a proposal

Álvaro Nolla[1], Tomás Recio[2]

[1]Universidad Autónoma de Madrid; alvaro.nolla@uam.es

[2]Universidad Antonio de Nebrija, Madrid, Spain; trecio@nebrija.es

Abstract

The connections between Mathematics and Art can be traced back to antiquity, where both disciplines have influenced and enriched each in several ways. It is with the arrival of the avant-garde and abstract art at the beginning of the 20th century that mathematics took on a greater and different relevance: as a source of inspiration and as a tool for artistic creation [1].

For some artists mathematics serves not only as a technical tool, but as a creativity source to explore new forms of expression. In these cases, approaching one of such works with a mathematical eye becomes an attractive and rich activity. The mathematization or reconstruction (often geometric) of the process that led the artist to complete his/her work encourages the search of deeper mathematics connections, further generalizations, and opens an analogous source of creativity now within the world of mathematics. In addition, the work of the artist thus reveals an instance of the beauty of mathematics.

In this direction, technological tools bring new possibilities to explore these connections and have the potential to create stimulating learning environments for both disciplines. In fact, the STEAM (Science, Technology, Engineering, Arts and Mathematics) educational approach provides an interdisciplinary framework where the circle of connections between mathematics and art is enhanced and enriched. For instance, a Dynamic Geometry Software (DGS) like GeoGebra[1] allows the creation an interactive mathematical layer over the artwork, as well as provides the appropriate scenario to explore the geometric constructions (see Figure 1, right). Together with the automated reasoning tools of GeoGebra Discovery [2], students can formulate and validate

[1] https://www.geogebra.org/

different mathematical conjectures and statements coming from the constructions, allowing them to deepen and broaden their knowledge of the mathematics that emerges from the work of art.

This study presents two examples of such STEAM activities around the work of two Dutch artists, Theo van Doesburg and Theo Jansen, with clear mathematical influences in the conception of their artworks. The objective of both proposals is to show how to use works of art as motivation for mathematical exploration using automatic reasoning and dynamic geometry.

The first proposal is centered on the painter Theo van Doesburg (Utrecht 1883 - Davos 1931), whose work began with the process of abstraction and geometrization within the *De Stijl* movement. We focus on one of his latest works, the *Arithmetic Composition I* (1929-1930).

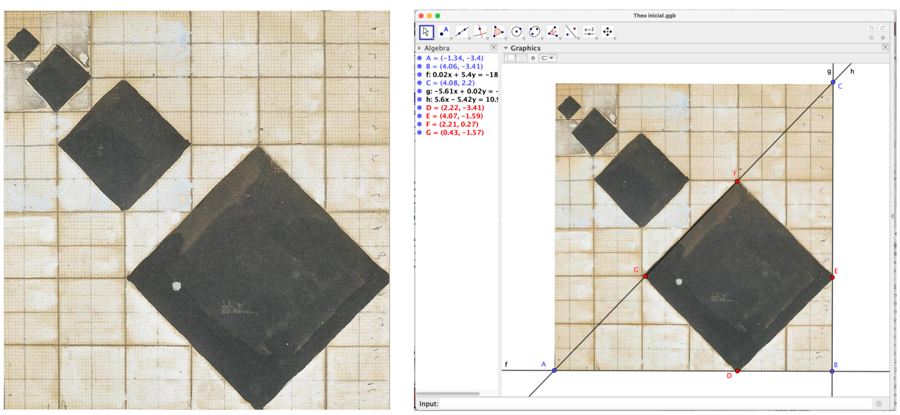

Figure 1. Theo van Doesburg Study for Arithmetic Composition[2] and its analysis in GeoGebra.

The painting shows a composition with four squares placed on one of the main diagonals (top-left to bottom-right). The author himself referred to his work in terms of "construction" rather than "composition" and was based on rational and geometric principles [3], so GeoGebra is shown to be a very suitable tool for its analysis.

The learning proposal for students starts from the creation in GeoGebra of the squares represented in the painting. The geometric properties that appear immediately when the painting is observed can be used for such construction (the symmetry of the squares with respect to one of the diagonals, and how the squares reduce the size of their sides by half). Added to this analysis

[2] https://en.m.wikipedia.org/wiki/File:Theo_van_Doesburg_study_for_Arithmetic_Composition.jpg

are the automated reasoning capabilities of GeoGebra Discovery to verify different conjectures that arise from the construction. For example, if we inscribe the quadrilateral DEFG in the isosceles triangle ABC (see Figure 1, right), must it be verified that AG=1/3 of AC? And does GF=1/3*AC? Is this the square with the biggest area that can be inscribed in the triangle ABC? How would the painter have known where he had to insert the vertices of the square so that such a property could be verified? Students will freely propose this type of conjectures and will use automatic reasoning tools to establish their veracity or generate the conditions so that they can be met. See Figure 3 (right) for an example.

The second part of our proposal focuses on the work of Theo Jansen (Scheveningen, 1948), an artist and kinetic sculptor who since 1990 has been building large figures imitating animal skeletons (autonomous machines) called *Strandbeest* (https://www.strandbeest.com). Conceived as new forms of life, Jansen's creations follow life cycle processes, his animals born, grow and reproduce, adapting to the weather and ground conditions. They are a fusion of art, engineering, and mathematics, being able to walk using the friction of the sand on Dutch beaches and the force of the wind.

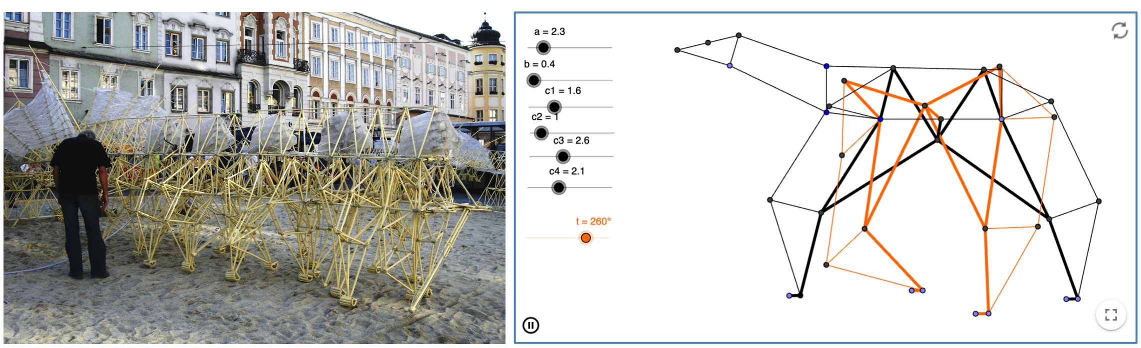

Figure 2. Picture of a *Strandbeest*[3] (left) and a GeoGebra applet[4] with a 4-leg replica (right).

The skeleton leg is a mechanism based on a system of 13 tubes joined in a linkage mechanism, which after a design process, Jansen improved to reach 13 holy numbers[5]. These particular

[3] https://en.wikipedia.org/wiki/Theo_Jansen#/media/File:Jansen-Strandbeest_crop.jpg
[4] https://www.geogebra.org/m/mNheeHTS by Markus Hohenwarter.
[5] https://espacio.fundaciontelefonica.com/theojansen/strandbeest-como-funcionan/

measurements make the animal remain at the same height and create the movement desired by the artist. See Figure 2 (right) for a 4-leg replica in a GeoGebra applet.

Our classroom proposal is based on the interdisciplinary work of Theo Jansen and the richness of linkage mechanisms within a STEAM education framework. It connects art, engineering and mathematics through linkages constructed in GeoGebra. Students will start creating simple mechanisms (originated in the industrial revolution at the end of the 18th century) and will explore different variations of the parameters in a design process (see for example the applet https://www.geogebra.org/m/fmhzvxfc). This modeling activity would give rise to Jansen's holy numbers and to the understanding of why the artist considers those legs to be more efficient.

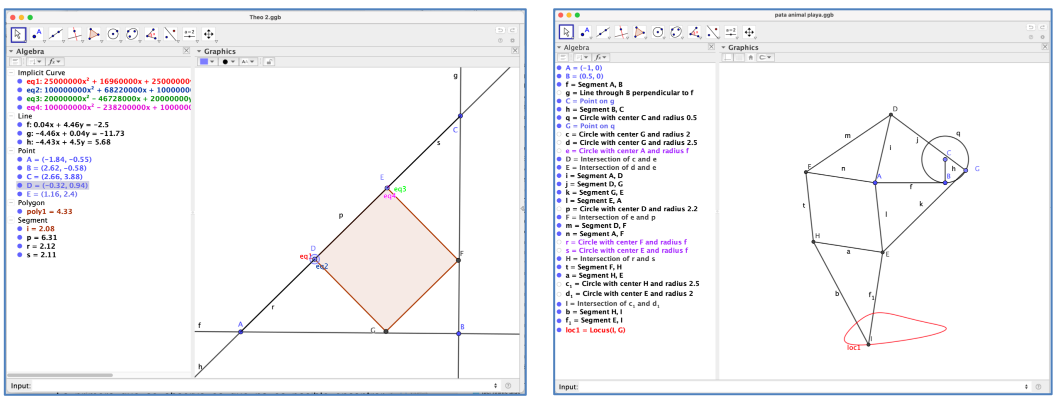

Figure 3. Examples of GeoGebra Discovery and GeoGebra applets with the constructions of the artists considered in the proposal.

In conclusion, we present *Arithmetic Composition I* and the *Strandbeest* as two examples where the use of automatic reasoning and dynamic geometry creates rich interdisciplinary STEAM learning contexts.

References

[1] Ibáñez Torres, R. (2023). *Las matemáticas como herramienta de creación artística*. Miradas Matemáticas. Ed. Catarata.

[2] Kovács, Z., Recio, T., Vélez, M.P. (2022). Automated reasoning tools with GeoGebra: What are they? What are they good for? In P. R. Richard et al. (eds.), *Mathematics Education in the*

Age of Artificial Intelligence, Mathematics Education in the Digital Era, vol 17. Springer, Cham. https://doi.org/10.1007/978-3-030-86909-0_2.

[3] Primm, D. (2001). Some notes on Theo van Doesburg (1883-1931) and his Arithmetic Composition I. *For the Learning of Mathematics,* 21(2):31-26.