Images and diagrams in David Eugene Smith's (1860 – 1944) works on the history of mathematics and on mathematics education

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This proposal continues our study of the historical development of educational innovations across times and places while being closely related to the MACAS axis "Historical and intercultural dimensions of studying mathematics". At MACAS2022, we presented the results of our study of Da Vinci's diagrams explaining his method of calculation of the area of the circle (for a paper based on this presentation see Freiman and Volkov 2023). We suggested that the drawings of Da Vinci may have inspired later educators to visualize the method of area calculation that can be found even in present-day textbooks and online resources. This aspect (i.e., visualization) was in the focus of our continuous analysis, since 2019, of the works of David Eugene Smith (1860–1944), a renown historian of mathematics and mathematics educator.

Smith's educational and professional backgrounds show a great diversity of areas of his interest and expertise. Originally, Smith obtained diplomas in law and liberal arts; his doctoral dissertation on the history of fine arts defended at Syracuse University in 1887 was titled "Polygnotos Ethographos: Polygnotos the Painter of Character"; as this title suggests, his thesis was devoted to the work of the ancient Greek painter Polygnotus (also spelled Polygnotos) active in the middle of the 5th century BCE (on this painter see, for example, Matheson 1995). In the late 19th century Smith travelled in Western Europe and collected there old books and artefacts; some of them were related to the history of mathematics. In 1890s Smith turned to mathematics, its history and its teaching; they became the key areas of his expertise while he worked as professor of mathematics in several American

educational institutions and the topics of his numerous publications. Numerous pages of his works on the history of mathematics contain portraits of ancient mathematicians, pictures of counting devices, reproductions of diagrams and of various tables from ancient and medieval mathematical texts, and other visual representations related to the history of mathematics. This attention to the "visual elements" might have been related to Smith's background in the history of fine arts. However, the authors of studies of Smith's work never investigated whether these pictures and reproductions were directly related to his narratives or some of them were added as mere decorations? In other words, were these pictures and diagrams systematically referred to and discussed in his texts, or were they related to his texts only loosely, and were added to draw attention of the readers?

In our contribution to the symposium we shall pay special attention to one particular work published by Smith in 1908; it had the following long title: *Rara Arithmetica: A Catalogue of the Arithmetics Written Before the Year MDCI with a Description of Those in the Library of George Arthur Plimpton of New York by David Eugene Smith of Teachers College, Columbia University*. One of the features of this "Catalogue" based on G.A. Plimpton's (1855–1936) collection (Donoghue, 1998) of early arithmetic treatises written before 1601 that deserves a special attention is the presence of numerous reproductions of diagrams, pictures, and reproductions of entire pages from the arithmetical works discussed in his book. According to Lambert L. Jackson, one of Smith's doctoral students whose dissertation of 1906 was devoted to the study of the Plimpton's collection with a special focus on its educational significance, the *Rara Arithmetica* was the 'most significant contribution to the history of mathematics treating specifically of the evolution of arithmetic during the sixteenth century' and contained 'extensive illustrations and reproductions' (Jackson 1939, p. 505).

For instance, Smith sometimes reproduced in his *Rara* entire pages of old books; for example, the multiplication table from a treatise of Boethius of 1488 occupied page 26, and often copied only a part of a page from an old book showing the materials that he considered especially interesting, as, for example, the representation of numbers "up to one million by means of the fingers and arms" reproduced from a treatise of Johannes Aventius (1477-1534) originally published in 1522 and reprinted in 1532 (Fig. 74 on p. 138). However, this large number of reproduced diagrams and pictures combined with relatively short commentaries made some pages of his book look as if they were taken from an exhibition catalog rather than from a scholarly publication.

Smith also paid special attention to the portraits of the mathematicians of the past mentioned in his works; our presentation will feature preliminary analysis of how these mathematicians were portrayed: how they were dressed? what were their body positions (for ex., sitting or standing when performing calculations, reading books, writing something or instructing their students, etc.)? If we take Smith's special attention to visual materials into consideration, it may explain why did he place a reproduction of a page of a textbook even on the first page of his own book.

We shall also compare Smiths approach to presentation of information about historical sources with the approaches found in the books on the history of mathematics published by other authors who worked before Smith; to do so, we shall compare his works with the *Arithmetical books from the invention of printing to the present time* published in 1847 by Augustus De Morgan (1806-1871) and with the *Rara Mathematica; or, A collection of treatises on the mathematics and subjects connected with them, from ancient inedited manuscripts* published in 1839 by James Orchard Halliwell-Phillipps (1820-1889) as well as with the books on the history of other disciplines (in particular, on the history of art).

Among other aspects that drew our interest were Smith's reproductions of the front pages of the medieval mathematical books which he provided without information of the contents of these books and thus did not allow the readers to understand what were the matters discussed in them and, in some cases, what was the

meaning of his reproductions; the pictures explaining the operations performed with computational devices were more useful, but not always. As for the portraits of mathematicians only a few of them were reproduced in the *Rara*: Pythagoras and his students (p. 46), Boethius and Pythagoras (pp. 82-83), Cardano (p. 194), Tartaglia (p. 277), Giovanni Francesco Peverone (pp. 291-292). Did Smith try to understand how the mathematicians were represented at their own time in order to make conclusions concerning their status in the society? Did he try to find pictures disclosing various social links, for example, the links that existed between mathematicians, as well as between mathematicians and their students? Or he simply picked up some "good looking" pictures and used them as mere decorations of his book, without any analysis of their contents? It can be conjectured that Smith's interest in the history of art and the research on this topic that he conducted in the very beginning of his academic career may have played an important role in his search for pictures and diagrams for his later publications related to the history of mathematics. We suggest that his choice of illustrations certainly deserves a further study; however, as far as we know, no works of modern historians of mathematics were focused on Smith's special interest in visual materials.

Concluding remarks

Not yet conclusive, our preliminary analysis helps to draw a possible agenda for further study which shall especially focus on the following topics:

(a) the depictions of mathematicians reproduced in Smith's works, especially the portraits of the "famous mathematicians of the past"; we shall discuss the criteria that he may have applied to the choice of these portraits, given that the depictions of certain mathematicians (for example, of Euclid) that circulated in the 19th and early 20th centuries most certainly were *not based* on their original depictions but represented the images invented by the later painters;

(b) depictions of mathematical and counting instruments, in particular, of the *abaci* and tables used for arithmetical operations (such as multiplication tables), and pictures showing the mathematicians using these instruments and tables;

(c) depictions of supernatural entities, goddesses/muses related to mathematics; for example, the picture of the goddess of arithmetic that appeared on

the opening page of some arithmetical manuals;

(d) reproductions of written records (e.g., of the records showing the procedure of multiplication of two numbers);

(e) depictions of two- as well as three-dimensional geometrical figures.

Finally, it will be explained why a more detailed comparative analysis of Smith's *Rara* is needed including not only the above-mentioned works by De Morgan and Halliwell-Phillipps but also by Asian historians who published books on the history of mathematics in the late 19th and early 20th centuries, in particular, with the books of Mikami Yoshio (1890-1960); a particular attention needs to be paid to their analysis of the diagrams found in pre-modern Chinese and Japanese mathematical texts.

In our concluding remarks we shall discuss the further work and the necessary methodological approaches; in this discussion we shall briefly analyze the recent works devoted to diagrams in pre-modern scientific traditions, e.g. Sidoli (2024).

References

De Morgan, A. (1847). *Arithmetical books from the invention of printing to the present time: being brief notices of a large number of works drawn up from actual inspection*. London: Taylor and Walton.

Donoghue, Eileen F. (1998). In search of mathematical treasures: David Eugene Smith and George Arthur Plimpton. *Historia Mathematica*, *25*(4), 359-365.

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Jackson, Lambert L. (1939). Addenda to *Rara Arithmetica*. *The American Mathematical Monthly*, Vol. 46, No. 8 (Oct., 1939), pp. 504-506.

Halliwell-Phillipps, James Orchard (ed.) (1839). *Rara mathematica; or, A collection of treatises on the mathematics and subjects connected with them, from ancient*

inedited manuscripts. London: J.W. Parker etc.

Matheson, Susan B. *Polygnotos and vase painting in classical Athens*. University of Wisconsin Press, 1995.

Sidoli, Nathan. The manuscript diagrams of Theodosios' Spherics. *Journal for the History of Astronomy*, vol. 55, issue 4 (2024), pp. 454–489.

Smith, David Eugene ([1908] 1970). *Rara Arithmetica; A catalogue of the arithmetics written before the year MDCI, with a description of those in the library of George Arthur Plimpton of New York*. Boston and London: Ginn and Company; fourth edition (1970) New York: Chelsea.