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Intend to Present/Attend: *Remotely*

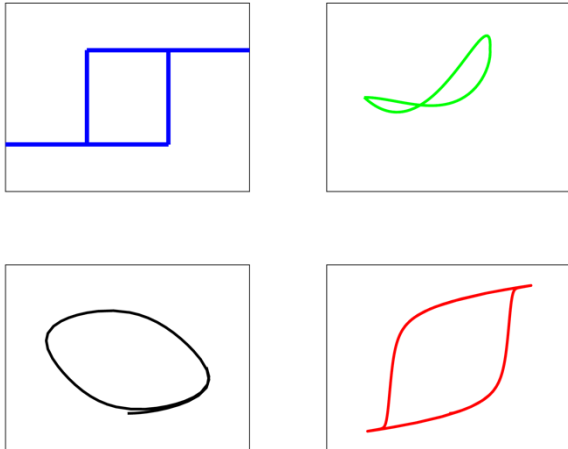**Type of communication**: Practical Example of Interdisciplinary Teaching and Learning

Figure 1. Shown are four examples of hysteresis loops, which were generated using the computing software MATLAB.

Title: A First Lesson in Hysteresis Loops

Description: Hysteresis is a phenomenon often visualized by a loop and is known to appear in a wide range of common processes like freezing-thawing, magnetism and in the performance of a thermostat. Figure 1 above illustrates examples of hysteresis loops. The mathematical definition of a loop is a closed curve; that is, a continuous line that starts and ends at the same point, which is the unifying commonality with all four curves in the above figure. As seen in figure 1, hysteresis loops appear in different styles and shapes. For example, the loop in green (top right) crosses itself and is known as a pinched hysteresis loop. The variety of styles and shapes of hysteresis loops is often dependent on the physical process it comes from. For example, the loop in blue (top left) is synonymous with the behaviour of thermostats.

There is no universally accepted definition for hysteresis, which makes teaching and researching hysteresis difficult. If taught, it would likely be in an upper year university level physics or engineering course, and would require students in the course to have (at a minimum) prerequisite mathematical knowledge of ordinary differential equations (ODEs). Therefore, teaching and learning hysteresis at an introductory and broadly accessible level is not common.

Previous attempts have been made to discuss hysteresis in a colloquial way [1-3], but they still require knowledge of ODEs. This symposium proposal presents novel and artistic approaches for teaching hysteresis to a general audience who have no prerequisite knowledge of mathematics beyond the high school level. These approaches include:

- describing hysteresis through fundamental characteristics that are unique to its looping behaviour,
- constructing physical hysteresis loops using yarn and crochet techniques,
- using colour to highlight parts of the hysteresis loop,
- exploring examples that are hysteretic with ones that are not,
- historical perspectives of the origin of hysteresis,
- animations illustrating the construction of hysteresis loops due to changes in a physical process; see [4] for example.

These approaches, when taught together, provide a broad conceptual understanding of hysteresis.

It can often be challenging to teach mathematical concepts (especially advanced ones) to students who have limited mathematical knowledge. It is the hope that the approaches presented in understanding hysteresis is a helpful example of teaching complex mathematical concepts in creative and approachable ways.

Connection to Macas theme (personal reflection): When I first began studying hysteresis, it was at a high level requiring graduate research and advanced mathematical analysis. It was also something I did not generally speak of aside from the small group of mathematicians and researchers who I also knew studied hysteresis. However, over the years, I have grown to regard hysteresis and its loops as aesthetically beautiful, appearing frequently in everyday activities and a phenomenon that can be appreciated by the general public. Having studied them for so many years, the image of the hysteresis loops evokes for me a sense of calm, comfort and familiarity; and represents a lifelong desire to keep learning about them and finding effective ways of sharing what they are. This continual quest is reminiscent of the spiritual and figurative motifs of circles and spirals associated to this year's MACAS symposium; as well as the more literal connection of looping behaviour in hysteresis.

References

[1.] A.N Chow, Hysteresis as an Authentic Mathematics Application, *For the Learning of Mathematics*, 43(1), 2023.

[2.] A.N Chow, K.A. Morris and G. Rabbah, Hysteresis and Stability, *SIAM Review*, 65(4), 2023. https://doi.org/10.1137/21M1420733

[3.] Morris, K. A., What is Hysteresis?. ASME. *Appl. Mech. Rev*. 2011; 64(5): 050801. https://doi.org/10.1115/1.4007112

[4.] Youtube video I co-created that explains hysteresis to a general audience https://www.youtube.com/watch?v=1o9DFKMOLb4