MACAS 2025 Proposal

Circles, Silence, and Noticing: Cultivating Mathematical Voice through Silent Teaching

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Mode of presentation: in person

> Mathematical facts are first guessed and then proved…such is the normal procedure…the student must be given some opportunity to do problems in which he first guesses and then proves some mathematical fact (Pólya, 1954, p. 160).

The proposal's objective is to exemplify how playing with numbers on a circle through a math routine of silent teaching unlocks potential opportunities for students to not only make observations about and verify properties of mathematical facts but to also notice connections between mathematics, arts, and sciences.

Much has been written about the importance of creating spaces in which learners can express and develop their mathematical thinking and sense making (Boaler, 2000; Schoenfeld, 1990). Yet, despite persistent calls for transformation, the use of student-centered learning in mathematics classrooms remains marginal (Avishai, Palatnik, Kolikant, 2025). Perhaps this is unsurprising, for established pedagogical strategies can indeed be tenacious (Boaler, 2002; Fry Nakar, & Zorn, 2025). In this paper, we explore the notion of silent teaching (Su, Wood, & Tribe, 2023) as a routine invitation for learners to attend to mathematical phenomena in ways that elicit students' noticing and conjecturing. It is reminiscent of Pólya's (1954) urging that mathematical thinking begins with the act of guessing—a kind of strategic imagination, nourished by careful observation. Silent teaching is not qualified by the absence of communication. Rather, its premise involves the deliberate withholding of teacher speech so that students may encounter mathematical phenomena freshly and develop their own questions and conjectures. In doing so, students develop their mathematical voices, not by being told, but by becoming attuned to mathematical regularity through their own experience.

We propose the silent teaching activity as a distinct form of a math routine to be added to teachers' professional repertoire. Unlike traditional teaching strategies, silent teaching shifts the focus away from "finding an answer" to a mathematics problem toward discovering what is going on. This routine invites students into the investigative and exploratory nature of mathematics, supporting them in their developing positionality as sources of knowledge and insight through their own questions, conjectures, and generalizations around patterns and structures they observe. As complementary to work on discovery-based approaches (Fielding, & Makar, 2022; Chase & Abrahamson, 2018), argumentation and mathematical agency (Mueller, Yankelewitz, & Maher, 2012), mathematical identity making (Fellus, 2019), the development of student mathematizing (Sfard, 2008), and low-floor, high ceiling mathematics (Gadanidis, 2012), silent teaching can be used as a math routine to develop students' mathematical voice and agency. We link to Ball (1993) who centralizes ongoing dilemmas in the three interrelated pedagogical principles of *representing content*, *respecting children as mathematical thinkers*, and *constructing community*. Ball (1993) provides a fascinating account of what student-driven mathematical conversation looks like when the teacher steps aside and opens up space for students to engage more fully and independently in mathematical dialogue. Building on Ball's (1993) work, we suggest that silent teaching positions students as capable of participating in mathematics both as learners and as producers of apt mathematical descriptions.

Drawing on a classroom vignette involving times tables in circles, we discuss how a math routine of silent teaching can lead to deeper student engagement and appreciation of mathematical ideas and their connections with the arts and sciences. We invite MACAS participants to our 'silent' talk to experience and surface potential pedagogical shifts.

Imagine a mathematics classroom where the teacher introduces a new mathematical idea without uttering a single word (see examples in Elementary Math at EDC, 2023). The students, already accustomed to this math routine (Cuoco, Goldenberg, & Mark, 1996), know that something important is unfolding. They watch intently, anticipating the next move, aware that their task is to observe, conjecture, and make sense of the unfolding regularity. In our case, our vignette involves times tables in circles (Mathologer, 2015). The teacher, without saying a word, sketches a circle with ten, equally spaced points and numbers them from 0 to 9. The teacher then continues labeling, indicating with ellipses that the pattern can extend indefinitely (see Figure 1).

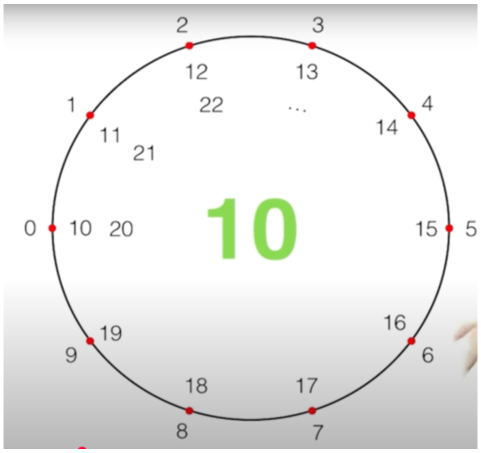

Figure 1: Creating a number Circle (Mathologer, 2015)

Next, the teacher writes x2 on the side of the circle, lists multiplication equations next to it, and draws lines in the number circle between each multiplicand and its product.

0 x 2 = 0 (0 on the number circle)

1 x 2 = 2 (draw a line between 1 and 2 on the number circle)

2 x 2 = 4 (draw a line between 2 and 4 on the number circle)

3 x 2 = 6 (draw a line between 3 and 6 on the number circle)

4 x 2 = 8 (draw a line between 4 and 8 on the number circle)

5 x 2 = 10 (draw a line between 5 and 0 on the number circle)

6 x 2 = 12 (draw a line between 6 and 2 on the number circle)

Throughout the activity, the teacher, without saying a word, uses gestures and facial expressions—such as a pensive look or a shrug to signal, when appropriate, openness to what comes next. Students all the while make guesses with regard to the regularity. After a few right guesses, the teacher raises her hand to indicate an invitation for students to take over. Hands shoot up, and the teacher gives the marker to one of the students, who comes to the board and continues the pattern while the others watch and continue to raise speculations about the regularity that is unfolding before their eyes (see Figure 2).

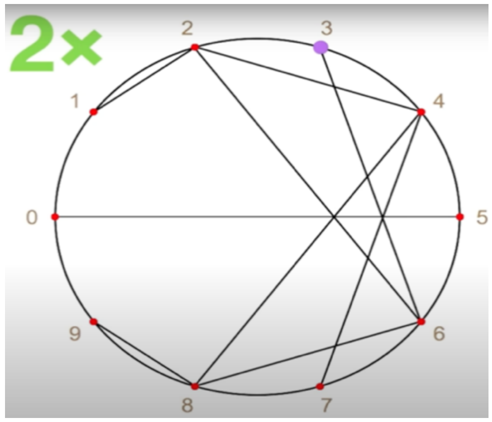

Figure 2: The multiples of x2 Circle with 10 points (Mathologer, 2015)

As the activity unfolds, students ask questions such as the 'What if not?' ones (Brown & Walter, 2005; Lavy & Bershadsky, 2003). In this context, developing a habit of student-generated questions supports the development of their mathematical voice. This habit "can help students to see a standard topic in a sharper light and enable them to acquire a deeper understanding of it as well" (Brown & Walter, 2005, p. 1). Such exploratory questions can be, but are not limited to:

- What if we use more points on the circumference?
- What if we use multiples of 3 instead of 2?
- What if we connect points according to different multiplication rules?
- What if we keep drawing more connections?
- What if we use the computer to draw infinite lines?
- What if we multiply by a fraction?

Exploring their own questions, the students can yield art such as the one seen in Figure 3.

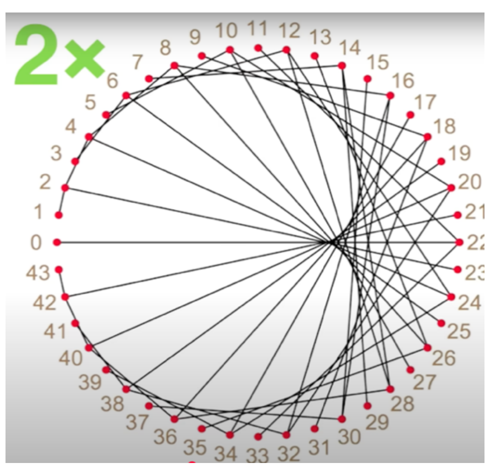

Figure 3: The multiples of 2 Circle with 43 points (Mathologer, 2015)

As students delve more deeply into their questions, they explore and appreciate some big ideas in mathematics (for e.g., cyclic symmetry, grouping, infinity, modular arithmetic and multiplication patterns, links between geometry and number theory). Through the visually compelling and mathematically captivating art—such as the cardioid in a x2 circle and the nephroid in a x3 circle—that materializes from their work, students can see that the relationship between the size of the multiplier and the number of petals follows the expression n-1 and showcases connection to the Mandelbrot set (see Figure 4).

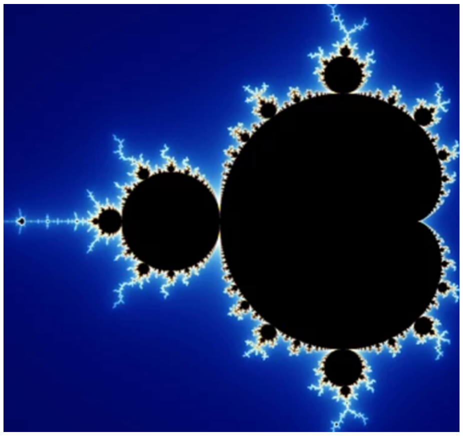

Figure 4: The Mandelbrot set

The teacher's silence in this routine is not empty, but filled with students' anticipation, appreciation, and mathematical thinking as the teacher listens to what is being said and affirms by gestures when students are getting closer and closer to identifying the regularity. During silent teaching moments and the discussion that follows, students become active participants in the construction of mathematical meaning by noticing, reflecting, conjecturing, and explaining. Along with other student-centered approaches, silent teaching challenges traditional notions of instruction and authority. As such, Su et al.'s (2023) invitation 'Dare to be silent' claims to be one of promoting student agency in mathematics classrooms. During the MACAS session, we will invite participants to experience silent teaching firsthand. Using simple materials (paper, pens, and circles), participants will engage in a silent mathematical activity, followed by a discussion on what they noticed, wondered, and learned.

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