Number 20 in Math, Art and Music

Sabine Segre, Achva Academic College, Israel

Sabine.segre@live.achva.ac.il

intended to be presented in person

On the occasion of 20 years of MACAS we will review where the number 20 appears and even connects between math, art and sciences.

Number 20 in Geometry and Art

To relate to circles, we will open with spirographs whose basic shapes are toothed circles – gears: The spirograph is a mechanical drawing tool consisting of 2 interlocking plastic gears, designed to create cycloids. The ratio of the numbers of the two gears determines the properties of the resulting rosetta, such as the number of leaves (Kang, Johnson, Lambert, & Davidson, 2020). Therefore, the mathematical properties of the spirograph determine the artistic outcome. To celebrate 20 years of MACAS, we will produce spirograph pictures with 20 leaves or cusps.

Moving from circles to spheres, we will see that a triangular pyramid can be constructed from 20 spheres, since 20 is a tetrahedral number.

Moving from circles to polygons we will discover the icosagon – a twenty-sided polygon. As a matter of fact, the icosagon was the shape of the foundation of The Globe, the outdoor theater used by William Shakespeare's acting company, as discovered in 1989 (Egan, 2004).

Moving to the third dimension, we will discover the icosahedreon, a polyhedron with 20 faces. The regular icosahedron is one of the five Platonic solids, and it is the shape of the Spinoza monument in Amsterdam. Plato identified the solids which he described with the five elements, and attributed water to the icosahedron, since it is closest to a sphere and therfore reminiscent of “fluidity”.

The truncated icosahedron is an Archimedian solid and the basic shape of a football, made from regular pentagons and hexagons. However, the faces of the football are not completely flat to render it more spherical.

Leonardo da Vinci depicted the icosahedron as well as the truncated icosahedron in Luca Pacioli's 1509 book *The Divine Proportion*. This book is in parts considered a plagiarism of Pierro della Francesca's book “De quinque corporibus regularibus”.

We will see a number of examples how the icosahedron and the truncated icosahedron inspired arts and architecture.

Escher’s 1948 lithograph *Stars* depicts an interlocking polyhedral structure, including an icosahedron.

The 20th Century

In the 20th century math and art meet (once again) due to new and quickly evolving possibilities of computer graphics.

Challenging questions led to beautiful illustrations, such as the task to draw 20 circles in the plane, all passing through the origin, but no two tangent at the origin. Additionally, except for the origin, no three circles pass through a common point.

Music

We will learn and experience what is special about the 20th Goldberg Variation and about Beethoven’s Piano Sonata No. 20 in G major.

Mathematical properties of 20

20 is a binomial coefficient, and gives therefore an answer to interesting combinatorial questions, of which we will present one.

Twenty is a regular number, as its only prime factors are 2 and 5. Regular numbers are numbers that evenly divide powers of 60 (or 30, which is equivalent). Relations between regular numbers can be beautifully displayed in Hasse diagrams (Weisstein).

The Mayan numeral system is a base 20 system, and the numerals can be considered artistic, since the Mayans had a very evolved calligraphy, so that it is sometimes said that they were writing pictures and painting words (Hudson & Henderson, 2018).

Curiosities

We will meet number twenty with the Simpsons, with Alice in Wonderland, solving the Rubik's cube and in a classical science fiction movie. We will also look for words which will have the value of twenty in the English and other languages, when assigning numerical values to letters.

Bibliography

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