**Enhancing Teachers' Geometric Reasoning with Paper Folding and GeoGebra**

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This proposal for an in-person presentation outlines ongoing research that examines the integration of paper folding and dynamic software (e.g., GeoGebra) in teaching geometric reasoning within a professional development course for elementary mathematics teachers. Our research question asks: How do paper folding and GeoGebra activities affect teachers' understanding of geometric reasoning and perceptions of teaching?

This study aligns with the MACAS 2025 theme, 'Circles of Resonance in Mathematics, Sciences, and Arts,' by employing circular and spiral patterns in learning activities that highlight the interconnectedness of mathematical concepts and teaching practices. Our research connects to this theme by combining paper folding and GeoGebra to teach symmetry. This combination reflects the theme's notion of spiraling circles, demonstrating how each resource—paper folding and dynamic software —builds upon the other to grow teachers' understandings and pedagogy in mathematics. For example, in the first session, we explored the properties of isosceles trapezoids and kites through paper folding and GeoGebra activities. In subsequent sessions, this approach is extended to other quadrilaterals: we investigate, compare, and define rhombuses & rectangles and squares & parallelograms. This iterative process is applied to different quadrilaterals, illustrating a spiraling approach to learning.

The main objective of our research is to investigate how the combined use of paper folding and GeoGebra influences elementary mathematics teachers' content and pedagogical content knowledge concerning symmetry. This study addresses the conference topic of curricular approaches by providing a practical example of how integrating paper folding and GeoGebra enhances geometric reasoning and instructional practices in mathematics teacher education. This integration notably supported teachers' strategies, encouraging them to adopt these approaches in their own classrooms to foster their students' mathematical reasoning.

**Preliminary Findings and Implications**: The ongoing study involves 20 teachers participating in a sequence of activities over a professional development course. Preliminary

findings indicate that the activities enhanced teachers' understanding of symmetry, with paper folding and GeoGebra proving to be influential tools for both learning and teaching. A particularly insightful component of the research involves teachers creating maps that show the hierarchical relationships among quadrilaterals based on their symmetrical properties. This mapping activity, which has sparked considerable discussion and insight among teacher participants, will be the focus of our presentation at the conference. The mappings are evidence of teachers developing a more nuanced understanding of geometric reasoning, which is linked to Van Hiele's Model of Geometric Thinking (Van Hiele, 1986).

Based on the study's results, we have identified the following three implications: 1) The integration of paper folding and GeoGebra has implications for enhancing teachers' knowledge of geometric concepts, particularly symmetry. These experiences influenced teachers' perceptions of how to effectively support their students' understanding and reasoning in geometry. 2) The study emphasizes the importance of purposefully sequencing investigations of quadrilaterals for teacher educators working with elementary teachers. The sequence of investigations exploring attributes, including symmetry, culminated in a mapping activity that demonstrated the hierarchical relationships among quadrilaterals. 3) The combination of paper folding and GeoGebra software encouraged a variety of strategies among teachers. These strategies enabled teachers to develop a deeper understanding of the relationships among quadrilaterals, fostering conceptual growth foundational for teaching geometric concepts. This finding is especially important as it demonstrates how paper folding and interactive software can be seamlessly integrated into the mathematics classroom to enhance both teaching and learning experiences. This research project offers new insights into effective strategies for teaching geometric reasoning, emphasizing the impact of integrated curricular approaches on teacher's professional development.