Proposal for MACAS 2025

Title: Rethinking Rectangle Diagonals: A Design-Driven Approach

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The proposal:

According to D'Ambrosio (1994), "to bring mathematics and science into context requires a deeper look into the place of education and of mathematics and science in modern societies". And it recognizes that cultural groups develop their own mathematical systems that are adapted to their specific needs and contexts as well. A group that stands out for its intense use of mathematics is that of graphic designers.

If we consider that graphic design is a topic that is well-liked and familiar to young students, we may propose learning trajectories relying on knowledge brought by the student, the knowledge shared and combined to resolve authentic problem situations common to this artistic field. In fact, when learning about graphic design, it is possible to observe many situations within the field's practice that mobilize a variety of mathematical contents such as geometric shapes, lines, area, proportion, spiral and angles.

Diagonal is one term that appears many times in design composition, a space in which visual elements are combined to create a unified design. And it seems to be in a state of tension. On one hand, it is readily observed that the word diagonal, in designs within a rectangular frame, is not necessarily used to refer to a line segment connecting two opposite vertices. On the other hand, it is possible to verify that designers do not use the term as defined in general dictionaries, which refers to any straight and sloping line that is neither horizontal nor vertical.

In graphic design, diagonals serve to structure and organize visual elements. Diagonals can also be used to create movement, depth, and balance within a composition. However, it is not clear whether they represent mathematical concepts and properties, nor whether designers are aware of these possible relationships. A preliminary

investigation of the subject revealed that diagonals in graphic design are rich in mathematical interpretations.

This communication is a theoretical analysis aimed at discussing mathematical thought considering the role of arts in understanding of mathematics in ordinary everyday situations. The investigation encompasses line segments within rectangles, concepts for the term diagonal and associated properties. Zazkis & Leikin (2008), Kabaca (2017) and Vízek, Samková & Star (2024) show references to the conception in which square is a particular instance within the broader class of rectangles or quadrilateral, and its diagonals are a particular case of the diagonal concept for these shapes. We are interested in establishing a reverse hierarchical relationship. We want to know what can be said if one thinks of a rectangle as a generalization of a square and diagonals of rectangles as generalizations of diagonals of square. This can be considered in the process of defining mathematical objects and in the context of teaching and learning mathematics (Burroughs & Burke, 2016). And it is considered by graphic designers. The possibility of generalization of a definition for diagonal, from squares to rectangles, is not unique, whether we consider equivalents or non-equivalents definitions. This is what we want to understand, having in mind designers' practices on image compositions.

The framework for analysis is the one introduced by Zazkis & Leikin (2007) and it is utilized to assess the didactic potential for the exercise of generalizing the concept of square diagonals. We consider accessibility and correctness, richness and generality of examples of generalizations based on cases analyzed in Zazkis & Leikin (2008), Kabaca (2017) and Vízek, Samková & Star (2024) and in design graphic materials, such as Hambidge (1967).

Findings revealed a favorable didactic potential for a work focused on generalizing the concept of square diagonal to rectangle. There are at least three important cases of generalization of the diagonal notion that are practiced by designers. One of them is particularly abundant in mathematical properties. The notion considers one diagonal as the usual one, while the other starts from another vertex perpendicularly to the first diagonal. This generalization can be seen in the works of the award-winning photographer Henri Georges Cartier-Bresson: https://expertphotography.com/henri-

cartier-bresson-composition/. This concept is related to many interesting properties. It allows us to create a notion of spiral that generalizes the golden spiral for any rectangle. The grid generated by these new diagonals has a remarkable relation with the rule of thirds grid, but if, and only if, the rectangle proportion is the square root 2. Other square roots play a role in this subject. It seems that designers are not totally aware of these properties.

Generalizing the concept of a diagonal from a square to a rectangle can be a pure mathematical exercise, based on observing and reproducing the properties of the diagonals of a square. We can identify many generalizations in this exercise. For example, in a square, one diagonal is the perpendicular bisector of the other. Applying this concept to a rectangle results in a different definition of diagonals. This mathematical formalization exercise can be expanded and enriched by studying or investigating the potential interest of this kind of grid creation for graphic design.

In this symposium I intend to examine further generalizations, present illustrative images, and discuss implications.

References:

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