Designing a Math Modeling Lesson about Zero-Waste Fashion Designs for Middle School Math Students

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The study addresses the MACAS topic of mathematical modelling and interdisciplinarity for studying and learning mathematics. The presentation is a practical example of interdisciplinary teaching and learning. Fashion design offers unique opportunities for students to engage in mathematical modeling. We analyze the planning process of a lesson for middle school students about creating zero-waste fashion designs. Zero-waste patterns result from designers' use of techniques to maximize the use of fabric so that nothing is leftover (Rissanen & McQuillan, 2024). This fashion design technique strives to achieve sustainability. The lesson provides students with an authentic design experience that prioritizes community-based outcomes and develops their environmental awareness. Mathematically, the lesson requires students to position pattern pieces without gaps and overlaps to maximize the use of the space and ensure zero-waste. Additionally, students apply reflection as a transformation to replicate a pattern piece and use the space efficiently. Students also must visualize how a combination of 2D shapes correspond to the construction of a 3D object when assembling their designs. Applying Doyle's (1988) conceptualization of a task, we ask: *What mathematical tasks did the teachers create to scaffold students' zero-waste fashion designs?* Our study focuses on the study and planning phases during a lesson study.

Methods and Data Sources

The data comes from a lesson study cycle with two middle school math teachers. Lesson study involves the process of studying instructional materials, planning, teaching, and reflecting on a research lesson (Lewis et al., 2006). In this lesson study cycle, the research team provided the teachers with three prototypes of lessons that apply the Human-Centered Design (HCD) approach. The teachers analyzed the lesson prototypes during the study phase. After examining the design, social justice, and geometry components of each prototype lesson, the lesson study team (researchers and teachers) used the zero-waste fashion design problem as a starting point for their research lesson. The lesson study team adapted the lesson to be taught in one of the

teachers' 7th grade math classrooms at a public school in the Midwestern part of the U.S. The 73-minutes lesson was meant to engage students in solving design challenge (Figure 1).

With your team, you will learn and apply the zero-waste pattern cutting technique to design a cross-body belt bag. **Your design should include at least one reflection to copy pattern pieces**. **Visualize** how each one of your 2D pieces will turn into a 3D model. You will begin by drawing a **rough sketch** of your design. Next, you will draw your **pattern layout** of how each piece will fit on a 25 in x 30 in sheet. Consider space for the seams and connections. Finally, **cut out** your pattern pieces and **assemble** your bag using tape.

Record a short video (3-min max) describing your design process. Explain how you used reflections and 2D-3D modeling to ensure zero waste. Also, share any other details you would like to incorporate or change in the future.

Figure 1. Task in the Zero-Waste Patterns Design Challenge

Theoretical Framework

The theoretical framework for the study is Doyle's (1988) conceptualization of a task where students use **resources** to perform **operations** to achieve a **product**. This framework has been applied to other research pertaining to students' geometry problem-solving. Herbst (2006). In this case, the design challenge of designing a cross-body bag was a mathematical problem with various tasks. As a modeling problem, there were various answers possible. The teachers designed various tasks to scaffold students' work in producing a solution.

Preliminary Results and Implications

When planning the research lesson, the teachers collaborated to break the design challenge into a set of mathematical tasks. These tasks were meant to scaffold student learning of the zero-waste pattern techniques. The teachers shared their own experience of designing a zero-waste cross body bag. As a result, they identified three math objectives: (1) visualizing how multiple 2D-objects can be manipulated to create a 3D-object, (2) optimizing area to reduce waste, and (3) using reflections efficiently to create congruent shapes. The teachers expected that the research lesson would enhance their students' connections with concepts pertaining to volume, a unit that would be taught afterwards.

The teachers designed the research lesson for students to decide how multiple 2D objects correspond to the construction of a 3D object. Students' visualization of the pattern pieces required an understanding of the properties of 2D objects (e.g., angles, sides, and curves).

Additionally, students had to visualize how to assemble the 2D objects to make a 3D object. The teachers sought to scaffold students' understanding of how to decompose a 3D object into a net or composition of 2D objects. One scaffold incorporated in the lesson to prompt students' visualization was encouraging students to brainstorm components of the cross-body bag that they wanted or needed. Then, students needed to translate those components into geometric figures. While this prompt was intended to support their students' mathematics thinking, it also considered the needs and wants of the students, a core idea in human-centered design. Additionally, the teachers provided students access to a set of nets of known 3D objects (e.g., prisms, pyramids). These nets were meant to aid the visualization from 2D to 3D as they could assemble and disassemble the nets. Upon completion of the pattern layout, the task of assembling the pattern is an operation that requires visualization.

The zero-waste aspect of the lesson was meant to encourage students to use the entire easel pad sheet (simulating fabric) provided to them. In the lesson, students must consider how to position each of their pattern pieces (2D-objects) to optimize the area and use every bit of the material. To scaffold this objective, the student worksheet included an example of a zero-waste backpack pattern layout. The teachers expected that by demonstrating how 2D objects could be arranged to use up all the space students would be inspired to do the same.

The teachers in the lesson study team wanted students to experiment with various operations such as paper folding to promote the use of reflections. Paper folding is an important operation that allows students to visualize a rigid motion that creates a congruent copy of a 2D object. The teachers added a prompt, encouraging students to consider how they can use reflections to replicate certain pattern pieces in their design. In the final part of the research lesson, the teachers added a task for students to ensure that the 2D pieces align to create a solid. The final task of cutting out and assembling their pattern layout was also a way for students to check if their ways to visualize the pattern work and accomplish a product: the fashion design of a cross-body bag.

The MACAS presentation will describe how the teachers finalized the plan for the research lesson and anticipated students' ideas, setting them up for implementing the lesson. Our work intends to contribute to an understanding of how teachers can use an interdisciplinary context to engage students in authentic mathematical modeling. The lesson also fostered students' visualization, an important skill (Presmeg, 2006). Integrating sustainable fashion design

to a math lesson can unlock opportunities to blend creativity, thoughtfulness and analytical thinking. The analysis of the design-based tasks for engaging students mathematical modeling can support future classroom implementation of design-based lessons.

References

Herbst, P. (2006). Teaching geometry with problems: Negotiating instructional situations and mathematical tasks. *Journal for Research in Mathematics Education, 37*(4), 313-347.

Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of Lesson Study. *Educational Researcher, 35*(3), 3-14.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. *Handbook of research on the psychology of mathematics education*, 205-235.