26th New Brunswick Math Competition (2009)

GRADE 9

HINTS

Remark:

Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Problems may have solutions other than the one suggested here.

Always try the problem before reading the hint!

1. Use the ordinary division algorithem. A quicher method is "casting out nines".
2. Since there are 270 seats in theatre 1, theatres 2 and 3 have a total of 530 = 800 - 270 seats. Now use two equations and 2 unknowns.
3. Solve $x + (x + 6) + (x + 12) + (x + 18) + (x + 24) = 100$
4. Notice that the sentence "Forty percent of the stalls are for ponies" is unnecessary information and that on Tuesday there were 750 animals in the barn.
5. If Bernard has x marbles, together they have $x + \frac{1}{2}x + (x + \frac{1}{2}x) + (x + \frac{1}{2}x + 10) = 109$ marbles.
6. The area of a triangle is $\frac{1}{2}$ base $\times$ height. The area of a rectangle is base $\times$ height. The base of the triangle and the base of the rectangle are the same length.
7. Sam received \$21 plus the bike for work of value $\frac{4}{7}$ (\$210 + the bike).
8. Estimate $\frac{365 \times 24}{5760}$.
9. $1 + \frac{2}{3} + \frac{4}{9} + \frac{2^3}{3^3} + \frac{2^4}{3^4} = \frac{3^4 + 2 \cdot 3^3 + 4 \cdot 3^2 + 8 \cdot 3 + 16}{3^4}$
10. $\frac{x \cdot 3}{(x + 3) + 7} = \frac{3}{2}$.
11. Simplify what is inside each of the brackets to a fraction. Then cancel.
12. If the son is x years old, $(x + 2) \cdot 5 = \frac{1}{2}(78 + 2)$.
13. A $4m \times 4m \times 4m$ hole has the volume of $\frac{4 \times 4 \times 4}{2 \times 2 \times 2} = 8$ $2m \times 2m \times 2m$ holes.

14. After $1\frac{1}{4}$ hours, the motor cycle had traveled $60 + \frac{1}{4} \cdot 60 = 75km$.

15. For the product of 3 integers to be odd, all three of them must be odd. There are 5 odd digits (1, 3, 5, 7, 9). So there are 5 choices for the first digit, 4 for the second digit, 3 for the third.

16. Consider the bottom right corner of the grid. The number in that position cannot be a 1 or 3, because the last column already contains both a 1 and a 3 nor can it be a 4 because the last row contains a 4. So it must be a 2. Continue like this to fill in the grid.

17. For two numbers a and b, $a^2 + b^2 - 2ab = (a - b)^2$.

18. $A = a^2, B = 4a^2, C = \frac{1}{2} \cdot 2a^2$.

19. After 1 minute, the horse has walked 75m and the fly is 75 meters ahead of the horse. Calculate how long it takes for the fly to return to the horse's nose.

20. Think of painting the houses in order. There are 5 possible colours for the first house, but only 4 possible colours for the next 3, since a house cannot be painted the same colour as the house painted just before it.

21. First consider the 1s digit. There are 10 2s used (2, 12, 22, ..., 92). 10 4s, 10 6s, 10 8s and 10 0s (10, 20, ..., 100). Then consider the tens digit. There are 10 2s (20 - 29), 10 4s, 10 6s, 10 8s, but only 1 0 (100).

22. Let h, c, d be the number of hens, camels and dromedaries respectively. Then, $2h + 4c + 4d = 4(0h + 2c + d)$.

23. All sides of the two boxes have to be painted except the bottoms of the two boxes and that part of the top of the bigger box that touches the smaller box.

24. Consider the last digits of $2^1, 2^2, 2^3, 2^4, ...$ and notice the pattern.

25. If we know the positions of 1, 3, 5 in the row (1st, 2nd, 3rd, 4th or 5th) the positions of 2, 4 and 6 are determined. There are $6 \cdot 5 \cdot 4$ ways to position 1, 3, 5 in the row.

26. A beautiful problem. First, calculate the edge length of the square (it is $\sqrt{2}$) so the half circle has radius $\frac{\sqrt{2}}{2}$. Notice that the shaded area is $\frac{1}{2}$ the area of the smaller circle less $\frac{1}{4}$ the area between the larger circle and the square.