26th New Brunswick Math Competition (2009)

GRADE 8

HINTS

Remark:

Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Problems may have solutions other than the one suggested here.

Always try the problem before reading the hint!

1. Check each choice in turn by subtracting 2 and then determine if the result is divisible by 3. You can just use the division algorithm, but an easy test for divisibility by 3 is this: A number is divisible by 3 if the sum of the digits is divisible by 3. (The sum of the digits of 4377 is 4+3+7+7=21.)
2. Three equations and unknowns works fine. A cleverer way to get to the answer is to notice that 100+90-110 is twice the number of boxes Daryl can pick in a day, so the answer is $\frac{1}{2}(100 + 90 - 110)$.
3. Since there are 270 seats in theatre 1, theatres 2 and 3 have a total of 530 = 800 - 270 seats. Now use two equations and 2 unknowns.
4. Solve $x + (x + 6) + (x + 12) + (x + 18) + (x + 24) = 100$
5. Notice that the sentence "Forty percent of the stalls are for ponies" is unnecessary information and that on Tuesday there were 750 animals in the barn.
6. Since one of the coins is a quarter, the other three have total value \$1.35. There is only 1 way to have three Canadian coins worth \$1.35.
7. The area of a triangle is $\frac{1}{2}$ base $\times$ height. The area of a rectangle is base $\times$ height. The base of the triangle and the base of the rectangle are the same length.
8. Sam received \$21 plus the bike for work of value $\frac{4}{7}$ (\$210 + the bike).
9. Estimate $\frac{365 \times 24}{5760}$.
10. If l is the length, and w the width of the chalk board, then $l = 2w$ and $l - 2 = w + 2$.
11. $1 + \frac{2}{3} + \frac{4}{9} + \frac{2^3}{3^3} + \frac{2^4}{3^4} = \frac{3^4 + 2 \cdot 3^3 + 4 \cdot 3^2 + 8 \cdot 3 + 16}{3^4}$
12. If the son is x years old, $(x + 2) \cdot 5 = \frac{1}{2}(78 + 2)$.
13. A $4m \times 4m \times 4m$ hole has the volume of $\frac{4 \times 4 \times 4}{2 \times 2 \times 2} = 8$ $2m \times 2m \times 2m$ holes.

14. Work backwards: If the dish was covered on day 16, it was only half covered on day 15 ....

15. Simplify what is inside each of the brackets to a fraction. Then cancel.

16. After $1\frac{1}{4}$ hours, the motor cycle had travelled $60km + \frac{1}{4} \cdot 60km = 75km$.

17. Consider the bottom right corner of the grid. The number in that position cannot be a 1 or 3, because the last column already contains both a 1 and a 3 nor can it be a 4 because the last row contains a 4. So it must be a 2. Continue like this to fill in the grid.

18. Count carefully! Half the rectangles have their sides parallel to the edges of the paper, half at a 45° angle to the paper.

19. One way to do this problem is to notice that the large triangle is twice as high and twice as wide as the small triangle. (Because the triangles are similar.) So (using $\frac{1}{2}$ base $\times$ height) the area of the large triangle is 4 times that of the small triangle.

20. For two numbers a and b, $a^2 + b^2 - 2ab = (a - b)^2$.

21. $A = a^2, B = 4a^2, C = \frac{1}{2} \cdot 2a^2$.

22. There are $4 \times 3 \times 2 \times 1$ ways of arranging the kids in a line, but some of those ways result in Brad and Cathy being next to each other. So, count the number of ways to arrange the kids with Cathy and Brad next to each other, then subtract.

23. All sides of the two boxes have to be painted except the bottoms of the two boxes and that part of the top of the bigger box that touches the smaller box.

24. Consider the last digits of $2^1, 2^2, 2^3, 2^4, ...$ and notice the pattern.

25. After 1 minute, the horse has walked 75m and the fly is 75 meters ahead of the horse. Calculate how long it takes for the fly to return to the horse's nose.

26. A beautiful problem. First, calculate the edge length of the square (it is $\sqrt{2}$) so the half circle has radius $\frac{\sqrt{2}}{2}$. Notice that the shaded area is $\frac{1}{2}$ the area of the smaller circle less $\frac{1}{4}$ the area between the larger circle and the square.