29th New Brunswick Math Competition (2011)

GRADE 9

HINTS

Remark:
Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Many problems have solutions other than the one suggested here.

Always try the problem before reading the hint!

1. Fractions. Be careful taking reciprocals. If $x = \frac{1}{2}$ then $\frac{1}{x} = 2$.
2. Let x represent the number of birds in the cage. Then $40 + .25\ (x - 60) = \frac{1}{3}x$.
3. Always simplify inside brackets first: $3 * 4 = 16 - 6 = 10$. Now, what is $10 * 5$?
4. Suppose the 4 consecutive integers are $x, x + 1, x + 2$, and $x + 3$. Then $2x = x + 3$.
5. The answer is the least common multiple of $4, 6$, and 9.
6. Compute the two ages for this year, then add them together. Now, father $= 2 \times$ me. Using the information about 10 years ago, (father $- 10$) $= 3 \times$ (me $- 10$).
7. If x is the number of questions Alex answered correctly, $4x - 2\ (15 - x) = 30$.
8. An important fact about triangles is that the sum of the lengths of any two sides is greater than the length of the third side. (Called the triangle inequality.) So, if two of the sides have length 4 and 5, the third side must have length less than 9.
9. If there are 3 (or more) red marbles there would be 21 (or more) blue marbles. But there are only 15 marbles in the box! What happens if there are 2 red marbles?
10. The question can be done by working backwards. John had \$1 after being in all 4 stores. So he must have had (\$1 + \$1) $\times$ 2 just after leaving the third store.

    "Guess and check" works well for this problem.
11. First calculate the cost of each chocolate bar – call it x. Then $8x = 3 \cdot \$4.00$. Jane spent $5x$ dollars and is owed $5x - \frac{8}{3}x$ dollars.
12. Find the least common divisor of 245 and 175.
13. Let s represent the number of sheep, g of goats, c of cows. Then $s + g + c - 4 = s$, $s + g + c - 6 = c$, and $s + g + c - 8 = c$. Simplify the equations and solve. Alternatively, think like this "three times all these animals but $4 + 6 + 8$ are all these animals". So there are 9 animals in total.
14. A hard counting problem. Count carefully perhaps drawing pictures of the different triangular shapes.
15. Let x represent the number of sports cars, y the number of sedans. Then $x + y = 12$ and $3x + 4y = 43$. Notice that whether the passengers or drivers are men or women does not matter.

16. Recall, distance = rate $\times$ time so time $= \dfrac{\text{distance}}{\text{rate}}$
If s is the length of each section, the total time taken is $\frac{s}{10} + \frac{s}{5} + \frac{s}{30} = \frac{1}{3}s$. The total distance traveled is $3s$.

17. A tough problem. A two digit number like 37 can (and should) be thought of as $3 \cdot 10 + 7$. Let a and b represent the digits of the distance traveled at noon. Then, at noon, the distance is $10a+b$, at 1:00 pm it is $10b+a$ and at 2 pm the distance traveled is $100a+b$. Since Daryl's speed is constant, $100a+b-(10b+a) = (10b+a)-(10a+b)$. So, $108a = 18b$ or $6a = b$. But a and b are digits so $a = b = 0$ or $a = 1, b = 6$. However, $0 \cdot 10 + 0$ is not a 2 digit number.

18. If x is the number of red faces on the second die, the number of ways for Mark to win is $1 \times (6-x) + 5x$ which must be 18, $\frac{1}{2}$ of 36. The surprising thing about this problem is that x does not depend on how many red and blue faces are on the first die.

19. It is easy to see that 8, 9, 10, and 12 cents can be made using the new coins.

20. Tough problem. Let A, B, and C be the number of holes/hour each of Albert, Bob and Carl can dig. Then $4(A + B) = 1$ and so on. Solve the three equations and unknowns.

21. Consider the last digits of $3^1 - 1,\ 3^2 - 1,\ 3^3 - 3,\ 3^4 - 1, ...$ and look at the pattern.

22. $x^2 + 2x^2 + 2011 = x(x^2 + x - 1) + (x^2 + x - 1) + 2012$.

23. Since there is a dog in 90 houses and a cat in 80 houses, there is both a dog and a cat in at least $70 = 100 - 10 - 20$. With a (dog and cat) in 70 houses and a rabbit in 75 houses, there must be 45 with a dog, a cat and a rabbit. With a (dog and a cat and a rabbit) in 45 houses, and a turtle in 65 houses, ...

24. $\text{guests} + \dfrac{\text{guests}}{2} \cdot (\text{guests} - 1) = 78$

25. Triangle ADE has $\frac{1}{3}$ the height and base of triangle ABC. So the area of triangle ADE is $\frac{1}{9}$. Sliding triangles DBF and ECG together we have a triangle where height and base are $\frac{2}{3}$ of that of triangle ABC, so the area is $\frac{4}{9}$. That leaves $1 - \frac{1}{9} - \frac{4}{9} = \frac{4}{9}$ as the area of the rectangle.

26. Here is neat way to think about the problem.
Imagine the entire track is turning about its center, so that from a bird's eye view the first cyclist stays still. The other cyclist then, from the bird's view, moves at 50 km/hr. Still from the bird's view, after an hour the second cyclist has gone around the track 5 times, so passes the first cyclist 5 times.

    A more straight forward method would be to work out that they pass at times $\frac{1}{10}$ h, $\frac{1}{5} + \frac{1}{10}$ h, $\frac{1}{5} + \frac{2}{10}$ h ...