28th New Brunswick Math Competition (2010)

GRADE 9

HINTS

Remark:
Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Many problems have solutions other than the one suggested here.

Always try the problem before reading the hint!

1. Fractions: Always simplify what is inside brackets first. Find a common denominator and add. Then cancel before multiplying.

2. Use the division algorithem.

3. Notice that the number in the box between that of 21 and that of 5 must be 11, since $21 + 11 = 32$. Find the numbers in the other boxes in terms of x and compute $y - x$.

4. Don't forget to count both end posts! You can check your method by doing a simpler but similar problem. For example, if the fence were 12m long it would clearly take 3 posts to build the fence. Draw a picture.

5. Set up the equation: $12x = 14(x - 4)$. Check your answer in the story, not just the equation.

6. Draw in the diagonals and count carefully. You will see that each vertex lies on 5 diagonals and each diagonal contains 2 vertices. So there must be $\frac{8 \cdot 5}{2}$ diagonals.

7. Draw a picture. Notice that each rectangle is $\frac{9}{2} \times 9$.

8. The biggest mistake for this type of problem is getting the fraction upside down. You want time per revolution, that is time/revolution. So you need to estimate $\frac{\text{seconds in a week}}{\text{revolutions in a week}}$.

9. Only birds have wings. Snakes don't have legs!

10. 2 boxes, each containing 5 oranges, weigh a total of 2×1.678. So $2 \times 1.678 - 2.278$ is the answer.

11. The radius of the smaller circle is $\frac{1}{2}$ that of the larger. You could let the larger circle have radius 1 unit, the smaller $\frac{1}{2}$ unit and use the formaula $A = \pi r^2$.

12. Check each number between 100 and 110 to see if its divisible by any of $2, 3, 5, 7$ (2 and 5 are easy !). Why do you only need to check these 4 numbers? You might like to investigate "The Sieve of Eratosthenes".

13. Before starting to read for the day, on Monday Grace had all the pages of the book yet to read, Tuesday she had $\frac{3}{4}$ of the pages yet to read, Wednesday $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$ of the pages yet to read and Thursday $\frac{3}{4} \cdot \frac{9}{16} = \frac{27}{64}$ of the pages yet to read. We are told that this is 81 pages.

14. If you have no idea how to do a problem, you might try easier problems and look for patterns.
$1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 6 \cdot 120, ...$
Ah Ha! $n!$ ends in at least one 0 if $n \geq 5!$

15. The key to this problem is to realize that the number of cars that entered the town and the number of cars that left the town that day must be the same.

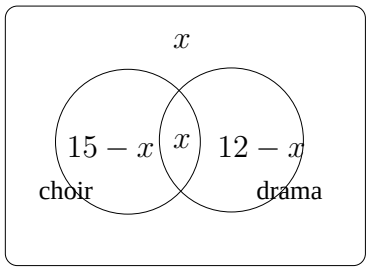
16. It is easier to calculate the areas of the three unshaded regions. It might be useful to draw a line through G parallel to AB and a line through F parallel to AD.

17. A cube of sand with side length $\frac{1}{2}$ meter $= 50$ centimeters weighs $2 \times 50 \times 50 \times 50$ grams, or $\frac{2 \times 50 \times 50 \times 50}{1000} = 250$ kg.

18. Suppose the 4 numbers are $a \leq b \leq c \leq d$. Then $a+b+c+d = 4 \cdot 24$, $a+b+c = 3 \cdot 20$ and $b+c+d = 3 \cdot 30$. You dont need to find b or c. You do need to find $b+c$.

19. Start by factoring: $120 = 2^3 \cdot 3 \cdot 5$. Now $2^3 = 8$,$2^2 = 4$ and $2 \cdot 3 = 6$ are the only digits that can be formed by multiplying two or more of the prime factors of 120. So the digits of any such number are $8, 3, 5$ or $4, 6, 5$. Each of these triples can be arranged in 6 ways.

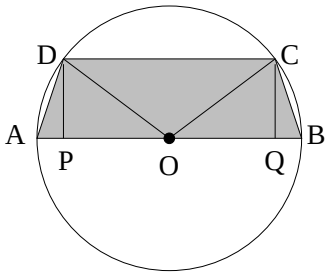
20. Use a Venn Diagram and let x be the number of students in both clubs.



21. Use two equations and two unknown. $\begin{cases} f + 45 &= 2(j - 45) \\ f - 45 &= j + 45 \end{cases}$.

22. If $\dfrac{1}{x^3 - 2x - 1} = -\dfrac{2}{3}$, then $x^3 - 2x - 1 = -\dfrac{3}{2}$ so $x^3 - 2x + 1 = \dfrac{-3}{2} + 2 = \dfrac{1}{2}$. (Trying to solve for x is hopeless!)

23. Add points P, O, Q to the diagram as shown. Note $OC = OB = 5$, and $PQ = DC = 8$. Compute PD.



24. Let r be the number of red computers and b the number of blue computers. The first box contains $r + \frac{1}{6}b$ computers. The other two boxes each contain $\frac{1}{2}(b - \frac{1}{6}b)$ computers, so $r + \frac{1}{6}b = \frac{1}{2}(b - \frac{1}{6}b)$ and we want $\frac{r}{r+b}$.

25. Let d be the distance between the cities, x be the speed at which the second car returns to city A. Using the formula time $= \frac{\text{distance}}{\text{rate}}$ we have $\frac{d}{60} + \frac{d}{60} = \frac{d}{90} + \frac{d}{x}$.

26. Find the length of the hypoteneuse to be 17 by Pythagora's Theorem. As suggested by the dots, join the center to the three vertices. Let r be the radius of the circle. Then the area of the big triangle is equal to the sum of the areas of the smaller triangle. $\frac{1}{2} \cdot 15 \cdot 8 = \frac{1}{2} \cdot 15r + \frac{1}{2} \cdot 8r + \frac{1}{2} \cdot 17r$.