29th New Brunswick Math Competition (2011)

GRADE 8

HINTS

Remark:
Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Many problems have solutions other than the one suggested here.

Always try the problem before reading the hint!

1. Fractions. Be careful.

2. Of course, you can just do the arithmetic, but it is easier to factor:
   $29 \times 71 + 29^2 = 29(71 + 29) = 29 \cdot 100$

3. The answer is the least common multiple of $4, 6$, and 9.

4. We want the number of ways to arrange A, B, and C without A in the middle. It is easy to just list them all.

5. Which of the given numbers (if any) has remainder 2 when divided by 3, and also remainder 2 when divided by 4?

6. Let x represent the number of birds in the cage. Then $40 + .25\,(x - 60) = \frac{1}{3}x$.

7. Suppose the 4 consecutive integers are $x, x+1, x+2$, and $x+3$. Then $2x = x+3$.

8. You could let x represent the number of boys in the class, y the number of girls. Then $x + y = 36$ and $\frac{x}{y} = \frac{4}{5}$.
   Or you might notice the class can be divided into 4 groups of 9 students, each group having 4 boys and 5 girls.

9. If there are 3 (or more) red marbles there would be 21 (or more) blue marbles. But there are only 15 marbles in the box! What happens if there are 2 red marbles?

10. The question can be done by working backwards. John had \$1 after being in all 4 stores. So he must have had $(1+1) \times 2$ dollars just after leaving the third store.

    "Guess and check" works well for this problem.

11. If x and y are the circumferences of the large and small wheels, we know $x = y + 1$ and $100x = 150y$. Find x and y, and use one of them to find the given distance.

    Here is another method. Imagine that the wheels are moving seperately, and each is moved 100 revolutions. The small wheel will be 100m behind the large wheel. The small wheel then moves 50 revolutions, and catches up. So, the distance traveled by 50 revolutions of the small wheel must be 100m. Since $150 = 3 \times 50$, the answer is 3×100.

12. Join the center of the circle to two adjacent vertices of the square. Use Pythagoras Theorem to see that the square has side length $\sqrt{2}$.

13. Find the least common divisor of 110 and 88.

15. It is easy to see that 8, 9, 10, and 12 cents can be made using the new coins.

16. A hard counting problem. Count carefully, perhaps drawing pictures of the different triangular shapes.

17. An important fact about triangles is that the sum of the lengths of any two sides is greater than the length of the third side. (Called the triangle inequality.) So, if two of the sides have length 4 and 5, the third side must have length less than 9.

18. If x is the number of red faces on the second die, the number of ways for Mark to win is $1 \times (6-x) + 5x$ which must be 18, $\frac{1}{2}$ of 36. The surprising thing about this problem is that x does not depend on how many red and blue faces are on the first die.

19. For the tens digits, we use each digit $1, ..., 9$ exactly 10 times. For the ones digits we also use each of $1, ..., 9$ exactly 10 times. For example, 1 is used as a 1s digit for $1, 11, 21, ..., 91$. (We also use 0 as a 1s digit, but 0 will not effect the sum of the digits).

20. Let x represent the number of sports cars, y the number of sedans. Then $x + y = 12$ and $3x + 4y = 43$. Notice that whether the passengers or drivers are men or women does not matter.

21. Here is neat way to think about the problem.
    Imagine the entire track is turning about its center, so that from a bird's eye view the first cyclist stays still. The other cyclist then, from the bird's view, moves at 50 km/hr. Still from the bird's view, after an hour the second cyclist has gone around the track 5 times, so passes the first cyclist 5 times.

    A more straight forward method would be to work out that they pass at times $\frac{1}{10}$ h, $\frac{1}{5} + \frac{1}{10}$ h, $\frac{1}{5} + \frac{2}{10}$ h, $\ldots$

22. $x^2 + 2x^2 + 2011 = x(x^2 + x - 1) + (x^2 + x - 1) + 2012$.

23. Tough problem. Let A, B, and C be the number of holes/hour each of Albert, Bob and Carl can dig. Then $4(A + B) = 1$ and so on. Solve the three equations and unknowns.

24. Since there is a dog in 90 houses and a cat in 80 houses, there is both a dog and a cat in at least $70 = 100 - 10 - 20$. With a (dog and cat) in 70 houses and a rabbit in 75 houses, there must be 45 with a dog, a cat and a rabbit. With a (dog and a cat and a rabbit) in 45 houses, and a turtle in 65 houses, ...

25. Since ABC is a right triangle, Pythagoras Theorem should come to mind,
    $AC^2 + AB^2 = BC^2$. $1 = \frac{\pi}{4} \cdot BC^2$, $2 = \frac{\pi}{4} \cdot AC^2$, $3 = \frac{\pi}{4} \cdot AB^2$.

26. A tough problem. A two digit number like 37 can (and should) be thought of as $3 \cdot 10 + 7$. Let a and b represent the digits of the distance traveled at noon. Then, at noon, the distance is $10a+b$, at 1:00 pm it is $10b+a$ and at 2 pm the distance traveled is $100a+b$. Since Daryl's speed is constant, $100a+b-(10b+a) = (10b+a)-(10a+b)$. So, $108a = 18b$ or $6a = b$. But a and b are digits so $a = b = 0$ or $a = 1, b = 6$. However, $0 \cdot 10 + 0$ is not a 2 digit number.