26th New Brunswick Math Competition (2008)

GRADE 8

HINTS

Remark:
Since this is a multiple choice competition, many problems can be done by checking the choices ("guess and check"). However the hints here are for solutions other than "guess and check" (with a few exceptions).

Always try the problem before reading the hint!

1. Fractions !
2. Calculate the two ages. Either use two equations and two unknowns or, since the word "age" is usually taken to mean an integer, consider the possible factorings of 24.
3. A rectangular box has a font and back, a top and bottom, and two ends. So this box has two sides of dimensions 8×12, two of 12×20 and two of 8×20.
4. The first *six* terms are $3, 2, 1, 3+2+1 = 6, 2+1+6 = 9, 1+6+9 = 16$.
5. Solve three equations and unknowns or, noting that the two weighings with the heaviest stone will produce the two largest weights, $80 + 63 - 49$ is twice the weight of the heaviest stone.
6. Since the first and third digits must be the same nonzero even digit, they are either 2, 4, 6, or 8. The middle digit can be any of 0-9.
7. $26 + 27 + \ldots + 49 + 50 = (1 + 25) + (2 + 25) + \ldots + (24 + 25) + (25 + 25)$
8. Letting x be the number of 2kg bags used, $2x + 5(2x) = 252$ and the answer is $x + 2x = 3x$.
9. The same parts of the path where Bob walks uphill on the way to work, he walks downhill on the way home. So, if on the way to work he walks u km up hill, l km on the level, and d km down hill, the time he takes to walk to work and then home again is

   $$\frac{u}{2} + \frac{l}{3} + \frac{d}{6} + \frac{u}{6} + \frac{l}{3} + \frac{d}{2} = 2$$

   This can be solved for $(u + l + d)$.
10. Calculate the dimensions of the shaded region:
    $6 + 6 - w = 10$ and $7 + 7 - l = 10$, where w and l are the width and length.
11. Divide the problem into 5 cases, that of $0, 1, 2, 3$ or 4 25 cent coins. Using no 25 cent coins, there are 11 ways to make \$1. (Think about the number of dimes used.) Using one 25 cent coin there are 8 ways to make \$1. Continue with 2,3, or 4 25 cent coins.

12. $7 \times 24 \times 60 \times 60$ can be computed. It can also be approximated:
$7 \times 24 \times 60 \times 60 \approx 7 \times \frac{100}{4} \times 60 \times 60 = 7 \times 15 \times 6 \times 10^3 = 105 \times 6 \times 10^3 \approx 6 \times 10^5$

13. The quickest way for someone to win 10 games is for Ahcène to win every game he plays.

14. From the first sentence, if 3 men are working, each cuts $\frac{72}{3 \cdot 3} = 8$ trees/hour. Thus if 5 men are working, each cuts 6 trees/hour.

15. A number is divisble by 3 if the sum of its digits is divisble by 3. Alterantively, a number is not prime if it is divisble by a prime number less than its square root. Thus we could check each number to see if it is divisble by 2,3,5,7, or 11.

16. There are three possibilities for the first position (2, 3, or 4) then two possibilities for the last position. The remaining 3 numbers can be arranged arbitrarily in the middle three positions.

17. $2008^{2008} > 1000^{2008} = 10^{6024}$ so 2008^{2008} has more (many more) than 6025 digits.
Also $1 \times 2 \times \ldots \times 2008 < 2008 \times 2008 \times \ldots \times 2008 = 2008^{2008}$

18. First think about the opposite problem: "In how many ways can 2 squares be removed so that the remaining region is _not_ connected."

    The 2 squares removed, must either be (2,5) or touch each other at a corner.

    Then consider "How many ways are there to remove 2 sqaures?"

    Finally, subtract.

19. Checking the 4 possible answers is the easiest way to do this.

20. Careful counting is the key. Here is one way to think about the possibilities. There are 5 sections to the chocolate bar. If the bar is cut into 2 pieces, one piece will have either 1 section or 2 joined sections.

21. Test each possible answer to see if it can be written as the sum of 3 squares. (Ex. $14 = 1 + 4 + 9$).

22. Numbers for which she will end up with 1 are 1, 100 and all two digit numbers whose sum of digits is either 1 (only 1 possibility) or 10.

23. Careful counting! Notice the 4 sided figures are the outer and inner squares as well as pairs of triangles that have an edge in common. Look for those common edges.

24. Use symmetry to see that the area is the difference of the area of a circle of radius 2 and a circle of radius 1.

25. It is easiest to think about which single digit integers are not used. $1 + \cdots + 9 = 45$, so we must omit 2 of the digits whose sum is 8.

26. The number of zeros is the number of factors of $10 = 5 \times 2$ in that number. The factors of 5 in the number $1 \times 2 \times \cdots \times 50$ can be counted by noting that the multiples of 5 have 1 or 2 factors of 5, with only 25 and 50 having 2 factors of 5. There are even more factors of 2.