29th New Brunswick Math Competition (2011)

GRADE 7

HINTS

Remark:

Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Many problems have solutions other than the one suggested here.

Always try the problem before reading the hint!

- 1. Solve the equation for x.
- 2. Of course, you can just do the arithmetic, but it is easier to factor: $29 \times 71 + 29^2 = 29 \times (71 + 29) = 29 \times 100$
- 3. You can solve two equations and two unknowns, but that is relatively hard. "their product is 24" suggests factoring 24.
- 4. $\frac{1}{3} \times 12 + .60 \times 10$
- 5. Which of the given numbers (if any) has remainder 2 when divide by 3, and also remainder 2 when divided by 4?
- 6. 50% more than the basic contract is $40 + .50 \times 40$ dollars.
- 7. To use as many different positive numbers as possible use the smallest possible numbers. So find the biggest n so that 1 + 2 + ... + n < 35
- 8. You could let x represent the number of boys in the class, y the number of girls. Then x + y = 36 and $\frac{x}{y} = \frac{4}{5}$. Or you might notice the class can be divided into 4 groups of 9 students, each group having 4 boys and 5 girls.
- 9. The answer is the least common multiple of 4, 6, and 9.
- 10. Notice there are two sizes of triangles to count.
- 11. The sum of the angles of a triangle is 180 deg.
- 12. If x and y are the circumferences of the large and small wheels, we know x = y + 1 and 100x = 150y. Find x and y. Use one of them to find the given distance.

Here is another method. Imagine that the wheels are moving seperately, and each is moved 100 revolutions. The small wheel will be 100m behind the large wheel. The small wheel then moves 50 revolutions, and catches up. So, the distance traveled by 50 revolutions of the small wheel must be 100m. Since $150 = 3 \times 50$, the answer is 3×100 .

- 13. If there are 3 (or more) red marbles there would be 21 (or more) blue marbles. But there are only 15 marbles in the box! What happens if there are 2 red marbles?
- 14. We want the number of ways to arrange A, B, and C without A in the middle. It is easy to just list them all.

- 15. If x is the number of questions Alex answered correctly, 4x 2(15 x) = 30.
- 16. An important fact about triangles is that the sum of the lengths of any two sides is greater than the length of the third side. (Called the triangle inequality.) So if two of the sides have length 4 and 5, the third side must have length less than 9.
- 17. Compute the two ages for this year, then add them together. Now, father = $2 \times \text{me}$. Using the information about 10 years ago, (father -10) = $3 \times (\text{me} - 10)$.
- 18. The question can be done by working backwards. John had \$1 after being in all 4 stores. So he must have had (1 + 1) × 2 dollars just after leaving the third store."Guess and check" works well for this problem.
- 19. Let x represent the number of birds in the cage. Then $40 + .25 (x 60) = \frac{1}{3}x$.
- 20. First calculate the cost of each chocolate bar call it x. Then $8x = 3 \times \$4.00$. Jane spent 5x dollars and is owed $5x \frac{8}{3}x$ dollars.
- 21. For the tens digits, Pierre wrote each digit 1, ..., 9 exactly 10 times. For the ones digits he also wrote each of 1, ..., 9 exactly 10 times. For example, he wrote 1 as a 1s digit for 1, 11, 21, ..., 91. (We also use 0 as a 1s digit, but 0 will not effect the sum of the digits).
- 22. Join the center of the circle to two adjacent vertices of the square. Use Pythagoras Theorem to see that the square has side length $\sqrt{2}$.
- 23. Let s represent the number of sheep, g of goats, c of cows. Then s + g + c 4 = s, s + g + c 6 = c, and s + g + c 8 = c. Simplify the equations and solve. Alternatively, think like this "three times all these animals but 4 + 6 + 8 are all these animals". So there are 9 animals in total.
- 24. Recall, distance = rate × time so time = $\frac{\text{distance}}{\text{rate}}$. If s is the length of each section, the total time taken is $\frac{s}{10} + \frac{s}{5} + \frac{s}{30} = \frac{1}{3}s$. The total distance traveled is 3s.
- 25. Since there is a dog in 90 houses and a cat in 80 houses, there is both a dog and a cat in at least 70 = 100 10 20. With a (dog and cat) in 70 houses and a rabbit in 75 houses, there must be 45 with a dog, a cat and a rabbit. With a (dog and a cat and a rabbit) in 45 houses, and a turtle in 65 houses, ...
- 26. If x is the number of red faces on the second die, the number of ways for Mark to win is $1 \ times(6-x) + 5x$ which must be 18, $\frac{1}{2}$ of 36. The surprising thing about this problem is that x does not depend on how many red and blue faces are on the first die.