27th New Brunswick Math Competition (2010)

GRADE 7

HINTS

Remark:
Since this is a multiple choice competition, many problems can be done by checking the choices. ("guess and check").

Usually, these hints suggest a method other than "guess and check". Problems may have solutions other than the one suggested here.

Always try the problem before reading the hint!

1. Fractions! First simplify inside the brackets by getting a common denominator and adding. Then cancel before multiplying.

2. Use the division algorithm.

3. Remember to evaluate what is inside brackets first, and that multiplication and division take precedence over addition and subtraction.

4. Don't forget to count both end posts! You can check your method by doing a simpler but similar problem. Say the fence were $12m$ long. Then it would take 3 posts to build the fence.

5. I must have had three 1 cent pieces. Ignoring those three 1 cent pieces, I had 4 coins worth 50 cents. Now we see that I had exactly one 25 cent piece and three more coins worth exactly 25 cents.

6. Set up the equation: $12x = 14(x-4)$. Check your answer in the story, not just the equation.

7. Draw in the diagonals and count carefully. You will see that each vertex lies on 5 diagonals and each diagonal contains 2 vertices. So there must be $\frac{8 \cdot 5}{2}$ diagonals.

8. Draw a picture. Notice that each rectangle is $\frac{9}{2}\,\text{cm} \times 9\,\text{cm}$.

9. Only birds have wings. Snakes don't have legs!

10. Notice what happens to any number when you multiply it by a multiple of 10. To compute $1 \times 2 \times \cdots \times 98 \times 99$ you multiply by 9 multiples of 10. How many zeros will be at the end of $1 \times 2 \times \cdots \times 98 \times 99$? The answer is quite a few more than 9!

11. Notice that the number in the box between that of 21 and that of 5 must be 11, since $21 + 11 = 32$. Find the numbers in the other boxes in terms of x to get $y = 69 + x$. A sneaky way to this problem is to notice that the possible answers do not depend on x. So let x be any number you like, say 0, and then compute y.

12. Use ratios: $5 : \frac{3}{4} = x : 6$ or $\frac{5}{\frac{3}{4}} = \frac{x}{6}$.

13. The biggest mistake for this type of problem is getting the fraction upside down. You want time per revolution which can be written as time/revolution. So you need to estimate $\frac{\text{seconds in a week}}{\text{revolutions in a week}}$

14. The radius of the smaller circle is $\frac{1}{2}$ that of the larger. You could let the larger circle have radius 1 unit, so the smaller has radius, $\frac{1}{2}$ unit and use the formaula $A = \pi r^2$.

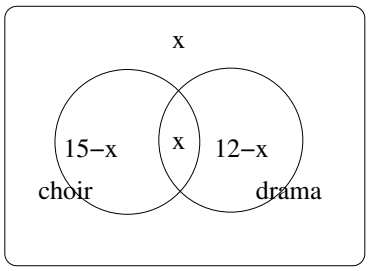
15. Check each number between 100 and 110 to see if its divisible by any of $2, 3, 5, 7$ (2 and 5 are easy !). Why do you only need to check these 4 numbers?

16. The key to this problem is to realize that the number of cars that entered the town and the number of cars that left the town that day must be the same.

17. 2 boxes, each containing 5 oranges, weigh a total of 2×2kg. So $2 \times 2 - 3.5$ kilograms is the answer.

18. Start by factoring: $120 = 2^3 \cdot 3 \cdot 5$. $2^3 = 8$, $2^2 = 4$ and $2 \cdot 3 = 6$ are digits. So the digits of any such number are $8, 3, 5$ or $4, 6, 5$. Each of these triples can be arranged in 6 ways.

19. Use a Venn Diagram and let x be the number of students in both clubs.



20. Notice that $(x^3 - 2x) = \frac{5}{3}$, so $(x^3 - 2x) + 1 = \frac{5}{3} + 1$. Solving for x is hopeless!

21. It is easier to calculate the areas of the three unshaded regions. It might be useful to draw a line through G parallel to AB and a line through F parallel to AD.

22. Suppose the 4 numbers are $a \le b \le c \le d$. Then $a + b + c + d = 4 \cdot 24$, $a + b + c = 3 \cdot 20$ and $b + c + d = 3 \cdot 30$. So $d = 4 \cdot 24 - 3 \cdot 20$, $a = \ldots$ You don't need to find b or c (and its not possible) but you will be able to find $b + c$.

23. Use two equations and two unknowns. $\begin{cases} f + 45 &= 2(j - 45) \\ f - 45 &= j + 45 \end{cases}$.

24. A cube of sand with side length $\frac{1}{2}$ meter $= 50$ centimeters weighs $2 \times 50 \times 50 \times 50$ grams, or $\frac{2 \times 50 \times 50 \times 50}{1000} = 250$ kg.

25. Before starting to read for the day, on Monday Grace had all the pages of the book yet to read, Tuesday she had $\frac{3}{4}$ of the pages yet to read, Wednesday $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$ of the pages yet to read and Thursday $\frac{3}{4} \cdot \frac{9}{16} = \frac{27}{64}$ of the pages yet to read. We are told that this is 81 pages.

26. Let r be the number of red computers, b the number of blue computers. The first box contains $r + \frac{1}{6}b$ computers. The other two boxes each contain $\frac{1}{2}(b - \frac{1}{6}b)$ computers, so $r + \frac{1}{6}b = \frac{1}{2}(b - \frac{1}{6}b)$ and we want $\dfrac{r}{r + b}$.