Proposed mixed species diameter growth model : an empirical study in Nova Scotia

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Abstract
There is a need for simplified diameter growth models for mixed species stands where parametrization is done based on available historical permanent sample plot data. In this paper, we propose a method that will allow modelling of the growth of multiple species together in the same stand. The main observation that drives our approach of combining multiple species in the same stand is that trees, regardless of the species, experience the same crown closure when on the same stand. This in turn yields a specific spacing for each species and spacing is the driving factor behind the NSDLF growth models. The paper presents the proposed method and empirical results from Nova Scotia PSP data. Sample sizes were small but there is enough evidence to warrant further investigation.

Keywords: growth model, multi-species, stand level

Résumé
Il existe un besoin de modèles de croissance de diamètre simplifiés pour les peuplements d'espèces mixtes où le paramétrage est effectué sur la base des données historiques disponibles sur les placettes d'échantillonnage permanentes. Dans cet article, nous proposons une méthode qui permettra de modéliser la croissance de plusieurs espèces ensemble dans un même peuplement. La principale observation qui sous-tend notre approche consistant à combiner plusieurs espèces dans le même peuplement est que les arbres, quelle que soit l'espèce, subissent la même fermeture de cime lorsqu'ils se trouvent sur le même peuplement. Cela donne à son tour un espacement spécifique pour chaque espèce et l'espacement est le facteur déterminant derrière les modèles de croissance NSDLF. Cet article présente la méthode proposée et les résultats empiriques des données PSP de la Nouvelle-Écosse. La taille des échantillons était petite mais il y a suffisamment de preuves pour justifier une enquête plus approfondie.

Mots clés: modèles de croissance, multi-espèces, niveau du peuplement

1. Introduction

In Nova Scotia, the Department of Lands and Forests (NSDLF) has been publishing forest research reports since the late 1980s. All told, over 90 reports have been published on topics mostly related to modeling growth and yield and the impact of management on productivity in forest operations in the province[1]. They have developed softwood and hardwood individual stand growth and yield models. These models are based on data collected from permanent sample plots (PSP) randomly distributed around the province and the data was collected over a 40-year period at 5-year intervals. The PSPs were set up in such a way that they would represent a variety of management regimes that are called treatment types (TRT). We can divide stands and their growth models into 5 treatment types presented in table 1.

[1] http://novascotia.ca/natr/library/publications/forestry-research.asp#research, visited November 2, 2014.

Table 1 - Treatment types

Treatment type	Description
1	Unmanaged natural stand
2	Pre-commercially thinned natural stand
3	Un-thinned plantation
4	Commercially thinned plantation
5	Commercially thinned natural stand

An unmanaged natural stand is a stand that has not had any silvicultural treatment applied to it since the last regeneration harvest and is growing naturally based on the land capability of where it is geographically situated. A pre-commercially thinned natural stand has been thinned to improve crop spacing and to improve growth, quality, and percentage of desirable trees (Canadian Council of Forest Ministers 2010). An un-thinned plantation has only been treated enough to ensure that the stand remains a plantation at its initial planting density without any other competing vegetation. In commercial thinning, the removal of trees is delayed until the stand has enough trees of marketable value so that releasing the stand not only gives more room for the remaining trees to grow but the trees being removed can be sold to create revenue (Canadian Council of Forest Ministers 2010).

We first present growth dynamics as they are modeled in N.S. for single-species individual forest stands. We then propose an approach for combining and growing two species together in the same stand and we test the model and present results.

2. Growth dynamics

The terms growth and yield are often used together but they are two separate terms. In this paper, growth refers to the change, over time, of average stand height, average stand diameter and crown closure. Height is always a function of age and site index (SI^{θ}) where site index is the dominant tree height in meters at age 50 years. Height does not depend on TRT and is not used as a stand variable in any of the growth models. Equivalently, we could have used height and site index and treated age as a derived variable. The yield of a stand refers to the volume of wood products that are available for harvest on a stand at any given time. This paper focuses on growth only.

The data collected and analysed from PSPs has been used to create equations that define the diameter growth of HW and SW stands for each of the treatment types presented in table 1 (NSDNR (1993b), O'Keefe and McGrath (2006)). Stands of treatment types 3 and 4 are pure SW stands as there aren't any HW plantations in Nova Scotia. Stands of TRT types 1, 2 and 5 can include SW and/or HW.

In most stand level growth models, site index (SI^{θ}) is an important factor. In any given stand, there may be a site index for SW and one for HW. In this work, growing a stand means using the equations to determine the values of the stand variables 5 years in the future. In order to simplify the discussion yet show relationships between variables more clearly, the following set of variables is used in stand growth and yield modeling.

cc_t crown closure fraction at time t
st_t stocking (% area forested) at time t
pct_t^{θ} percentage of total basal area of the stand at time t that is species type θ
d_t^{θ} stand quadratic mean diameter at time t for species type θ (centimeters)
ha_t height age for stand at time t

Crown closure is defined as a measure of the proportion of the ground covered by the vertical projection to the ground of the crown of all trees on the stand (Canadian Council of Ministers 2010). Stocking can be less than 100% if the stand has open areas such as swamps and rocky outcrops that prevent tree growth. The basal area of a tree is the round surface area of the stem of a tree when the tree is cut at 1.37m from the ground that is known as breast height (Canadian Council of Forest Ministers 2010). So, a tree with a diameter at breast height of 10 cm would have a tree basal area (TBA) of 0.007854 m2 (π·r2). The total basal area of a stand is the sum of the TBA of all the trees on a stand. The quadratic mean stand diameter of the trees on a stand is a basal area weighted average. More precisely, it is the diameter of a tree of average basal area (Gove 2003). For example, if a stand has 150 m2 of basal area and 2800 trees, the stand will have a quadratic mean stand diameter of 26.12 cm ($2 \times \sqrt{(150/2800)/\pi}$). Here, quadratic mean stand diameter is referred to as diameter (d) and the total stand basal area is referred to as basal area (BA).

The t subscript in the variables above represents time. NS DNR did their data collection, at 5-year intervals, over a 40-year period. Therefore, the growth equations are built for 5-year intervals and t+5 means 5 years have elapsed.

The age discussed so far refers to the height age. It`s the actual age of the stand since it`s last regeneration harvest and is the one used to calculate the height of the dominant trees on the stand using the site index curves. Diameter age or fake age (fa_t^{θ}) is used when calculating diameter growth for certain types of managed stands. Fake age isn't the actual age of the stand but rather an age that represents the current state of the stand which may be the result of a series of silvicultural operations that modified diameter and spacing since the last regeneration harvest. With either fa_t^{θ} or ha_t, we can write $age_t + 5 = age_{t+5}$.

Once growing stock is established on the stand, growth of the stand can be calculated using the models in this section. The growth models discussed in this section can be divided into 5 sets based on the stand characteristics and they are presented in table 2. For stands of TRT types 2, 3, 4 and 5, spacing (SP_t^{θ}) and species are used to determine which set of equations to use for growth calculations. SP_t^{θ} is calculated using equation (1).

$$SP_t^{\theta} = \frac{BA_t^{\theta}}{3.1415926 \times \left(\frac{d_t^{\theta}}{200}\right)^2} \qquad (1)$$

Functions f_i presented in this section and the next are taken from NS DNR research reports and can be found in Comeau (2011).

Table 2 – Sets of forest stand growth equations

Set 1	TRT = 1 (non-species specific)
Set 2	TRT types 2, 3, 4, 5 with $SP^S \geq$ 3.1m (softwood only)
Set 3	TRT types 2, 4, 5 with SP^S < 3.1m (softwood only)
Set 4	TRT = 3 with SP^S < 3.1m (softwood only)
Set 5	TRT types 2, 5 (hardwood only)

Set 1: This set includes all unmanaged natural stands which are the easiest growth to calculate as they are always at 100% crown closure. Therefore, the number of trees on the stand depends only on d_t^θ which is a function of ha_t and SI^θ. Growing the stand is simply a matter of adding 5 years to the current age (ha_{t+5}). All other stand variables are dependent on the new age ha_{t+5}. Equations (2), (3) and (4) are used to calculate d_t^θ which is required for yield calculations.

$$dht_t^\theta = f_1(ha_t, SI^\theta) \quad (2)$$
$$aht_t^\theta = f_2(dht_t^\theta) \quad (3)$$
$$d_t^\theta = f_3(aht_t^\theta, SI^H) \quad (4)$$

Set 2: This set includes softwood stands of TRT types 2, 3, 4 and 5 that have $SP_t^S \geq 3.1m$. Rather than one equation, calculating d_{t+5}^S for this set requires a sequence of preliminary values which are calculated with the equations presented in Comeau (2011). Using d_t^S and the appropriate maximum stocking line, $maxtrees_t^\theta$ is obtained. The stand crown closure, SI^S, d_t^S and $maxtrees_t^\theta$ are used to calculate d_{t+5}^S.

$$d_{t+5}^S = f_4(d_t^S, SI^S, maxtrees_t^\theta, cc_t) \quad (5)$$

Set 3: This set includes softwood stands of TRT types 2, 4 and 5 that have SP_t^S < 3.1m. We calculate a fake age (fa_t^S) corresponding to the observed spacing, diameter and site index and it may differ from the actual age of the stand (ha_t^S). For a given site index and spacing, f_5 is the inverse of f_6.

$$fa_t^S = f_5(SP_t^S, d_t^S, SI^S) \quad (6)$$

5 years of growth are added to fa_t^S and the new diameter is calculated with fa_{t+5}^S.

$$d_{t+5}^S = f_6(fa_{t+5}^S, SP_t^S, SI^S) \quad (7)$$

Set 4: This set includes softwood stands of TRT type 3 that have $SP_t^S < 3.1m$. Plantations (TRT=3) are like natural stands in that the stand grows differently if it has been thinned but plantations don't contain any hardwood so there is only one set of growth equations. The diameter growth of a plantation with SP_t^S < 3.1m cannot be calculated directly with one equation. Rather d_{t-2}^S and d_{t+3}^S are calculated and the difference between the two values is the diameter growth for the next 5 years.

$$d_{t+3}^S = f_6(ha_{t+3}^S, SP_t^S, SI^S) \quad (8)$$
$$d_{t-2}^S = f_6(ha_{t-2}^S, SP_t^S, SI^S) \quad (9)$$

$$d_{t+5}^S = d_t^S + d_{t+3}^S - d_{t-2}^S \quad (10)$$

Set 5: This set includes all hardwood stands of TRT types 2 and 5 regardless of spacing. The hardwood diameter growth model does not explicitly depend on spacing therefore there is only one diameter growth model (eq. 11).

$$d_{t+5}^H = f_7(d_t^H, BA_t^H, SI^H) \quad (11)$$

The last step in the process for any single species stand of treatment types 2, 3, 4 and 5 is to calculate crown closure (cc_{t+5}) which does not have a species indicator. Using d_{t+5}^θ and the appropriate maximum stocking line, $maxtrees_{t+5}^\theta$ is obtained. Using equations (12) and (13), $TFREQ_{t+5}^\theta$ and cc_{t+5} are calculated.

$$TFREQ_{t+5}^\theta = f_8(d_{t+5}^\theta) \quad (12)$$

$$cc_{t+5} = \frac{TFREQ_{t+5}^\theta}{maxtrees_{t+5}^\theta} \times 100 \quad (13)$$

The next section discusses how the growth of two species in a single stand is modeled in this work.

3. Proposed Method for Growing a Mixed-species Forest Stand

Up to this point, all growth models presented are those developed by NS DNR and they are divided into sets for easier presentation. In this section, we propose a new method for combining growth models for more than one species into a single stand mixed-species growth model. It follows directly from the models presented in the previous section and some results of applying this modelling approach to typical stands found in Nova Scotia are presented in the next section.

The previous section divided the growth models into 5 sets and, except for set 1 which deals only with natural unmanaged stands, they are all single species diameter growth models. In set 1, diameter and height growth depend only on age and site index of the stand. In that case, any information related to species mix on the stand will be used solely for volume calculation and are not covered by the modelling approach proposed here. Sets 2 to 5 deal with managed natural stands and plantations and are divided according to species and/or spacing. With single species stands, the equations in the previous section can be applied directly without considering the mixed-species approach proposed here. When dealing with a mixed-species stand, the sets of growth equations in the previous section all apply as presented but we work on the assumption that each species in a multi-species stand grows as if it were alone on a smaller stand. In this section, we explain the proposed approach of how to calculate the respective spacing for each species type when they are growing together on the same stand, how to grow each of them according to their respective growth equations and how to put them back together to calculate the new values of the multi-species stand variables.

The main observation that drives our approach and the assumption above is that trees, whether softwood or hardwood, experience the same crown closure when they are on the same stand. This crown closure creates an effective spacing for each species. This then can drive all the models. In the next section, the proposed methodology is illustrated by way of some examples. The proposed approach can be summed up by the following steps:

- Step 1: Calculate the fraction of the stand covered by each species type.
- Step 2: Calculate the respective spacing for each species type. This is done since crown closure is the same for each species type.
- Step 3: Given the spacing for each species type, calculate diameter growth using the respective growth models.
- Step 4: Combine the two species types to calculate the stand characteristics of the new stand.

What follows is a detailed explanation of each step with accompanying equations where appropriate.

Step 1: Calculate the fraction of the stand covered by each species type

The procedure starts with the calculation of the maximum stocking basal area for each species type, were each a pure single species stand, using the function below:

$$BAFULL_t^\theta = maxtrees_t^\theta \times 3.1415926 \times \left(\frac{d_t^\theta}{200}\right)^2$$

Using the equations for $maxtrees_t^\theta$ published by NS DNR (NSDNR (1993b), O'Keefe and McGrath (2006)) which depend on d^θ, we determine that a pure single-species SW stand has approximately twice as much BA as a pure single-species HW stand ($BAFULL^S$ = 60m^2 vs. $BAFULL^H$ = 30m^2). With this difference of maximum BA for the two species, the fraction of the ground area of the stand covered by each species type in a mixed-species stand isn't the same as pct^S and pct^H, which are the percentages of stand total basal area for each species type. Therefore, we define $fraction^\theta$ as the fraction of ground area of the stand covered by species θ to yield pct^θ and we assume that this fraction remains unchanged until a silvicultural treatment changes it. This assumption is based on the fact that even-aged stands are defined as stands that don't have advanced regeneration that can come up underneath the canopy to fill any openings in the stand (Smith 1997). We require that

$$\sum_\theta fraction^\theta = 1$$

With two species, the solution is easy: $fraction^S$ is the solution to

$$pct^S = \frac{fraction^S \times BAFULL^S}{fraction^S \times BAFULL^S + fraction^H \times BAFULL^H}$$

where pct^S is given as a stand variable. The solution is:

$$fraction_t^S = \frac{1}{\left\{\left(\frac{1}{BAFULL_t^H}\right) \times \left[\left(\frac{BAFULL_t^S}{pct_t^S}\right) - BAFULL_t^S\right]\right\} + 1}$$

and $fraction_t^H = 1 - fraction_t^S$, which are then used to calculate spacing in step 2.

<u>Step 2:</u> Calculate spacing between the trees

In this step of the proposed approach, we take advantage of the fact that crown closure is the same for both species on the same stand. Given the number of trees on the stand for each species and the area of the stand covered by each species, we can calculate the spacing between SW and HW trees as they are perceived by the individual trees on the stand. It is this spacing that is used in the diameter growth models from the previous section.

If crown closure < 100%, the actual number of trees on a pure single-species stand ($TFREQ_t^\theta$) is a proportion of $maxtrees_t^\theta$ according to the following relationship:

$$TFREQ_t^\theta = maxtrees_t^\theta \times cc$$

On a mixed species stand, a fraction of the stand is covered by each species type so we adjust the number of trees as follows:

$$TFREQ_t^\theta = fraction_t^\theta \times maxtrees_t^\theta \times cc$$

Given that, in this work, a stand is defined as being one hectare in size and that a hectare is 10,000m^2 (100m × 100m), we calculate the spacing for each species as follows:

$$SP_t^\theta = \sqrt{\frac{10000 \times fraction_t^\theta}{TFREQ_t^\theta}}$$

Given the spacing for each species type, we proceed to step 3 where diameter growth is calculated.

<u>Step 3:</u> Grow the species types according to their respective growth equations

What follows is an explanation of the adjustments that need to be made to each set of equations in the previous section for combining SW and HW in the same stand. Equations from set 2 are used when $SP^S \geq 3.1$m but the growth equations do not explicitly depend on spacing. In this case, the spacing calculated above is only used to determine whether the equations from set 2 should be used. The same goes for set 5. In sets 3 and 4, the diameter growth equations explicitly depend on SP^S and SP^H. Once the spacing for each species has been calculated as described in the previous step, growth equations in sets 3 and 4 are applied as described in the previous section to determine diameter growth hence the new diameters for SW and HW. With spacing for both species types known, the appropriate set of growth equations is chosen and SW and HW diameter growth Δd_t^θ is calculated.

4. Methodology used for testing proposed model

We test our model by using a paired t-test with the following hypothesis: $H_0: \mu_D = 0$ and $H_1: \mu_D \neq 0$ with test statistic $t = \frac{\bar{X}_D - \mu_D}{s_D/\sqrt{n_D}}$ which is student-t distributed with $\nu = n_D - 1$ degrees of freedom. In our case, μ_D is the average of the differences in pairwise values of the predicted five-year diameter change (Δd_t^{θ}) and the actual five-year diameter change, and n_D is the number of paired differences.

5. Empirical results

Empirical data was collected by the Nova Scotia Department of lands and forests by remeasuring trees at five-year intervals on permanent sample plots covering the entire province. Tree species were divided into 4 species categories: softwood (SW), tolerant hardwood (TOL), intolerant hardwood (INTOL) and aspen (ASP). Average diameters and basal area values were calculated for each sample plot, for each species class and for each remeasurement. Site index values were available in the database.

After cleaning the database and removing zeros and missing data, the remaining sample sizes were surprisingly small. No viable information could be kept for the ASP species class; therefore, it is not presented here. For the other three species classes, table 3 presents sample data.

Table 3 – Sample data

Species	Sample size	μ_D	Std. dev.	$t_{0.05,51}$	$t_{calculated}$	p-value	Accept H_0
SW	52	0.664	1.416	2.008	3.382	0.001	No
TOL	31	0.679	1.044	2.042	3.618	0.001	No
INTOL	48	0.196	0.674	2.012	2.016	0.049	Yes

Our small sample sizes prevent us from stating that we have significant statistical evidence to accept or reject our null hypothesis. However, in the case of intolerant hardwoods, there is enough evidence to warrant further investigation and the proposed methodology may be more appropriate for certain types of trees.

6. Conclusion

Development of this mixed-species model is based on the idea that all trees on a stand, regardless of species type, experience crown closure as being the same. The examples presented in this paper rely on the fact that crown closure is directly related to spacing between the trees on a stand and, in the growth and yield models published by NS DNR, spacing is the major driving force of individual SW and HW diameter growth models.

Evidence shows that the proposed method of growing mixed species stands may have some merit for specific species classes, but further investigation is warranted. In particular, the sample size needs to be increased.

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