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Combining two species in an individual forest stand growth model

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Abstract : *In Nova Scotia, the department of natural resources has developed softwood and hardwood stand level growth and yield models but nothing exists presently to model them together in the same stand. In this paper, we propose a method that will allow modelling of the growth of two species together in the same stand. The main observation that drives our approach of combining softwood and hardwood in the same stand is that trees, regardless of the species, experience the same crown closure when on the same stand. This in turn yields a specific spacing for each species and spacing is the driving factor behind the growth models. By growing each species based on the amount of space it covers on the stand, each species is grown as if it was alone on a smaller stand and then put back together on a bigger stand. The paper presents the proposed method and some examples of the method applied to typical mixed-species stands from Nova Scotia.*

Keywords: *growth model, multi-species, stand level*

Résumé :*En Nouvelle-Écosse, le département de ressources naturelles a développé des modèles de croissance et production des conifères et feuillus au niveau du peuplement mais rien n'existe présentement pour modéliser les deux espèces ensemble dans le même peuplement. Dans cet article, on propose une méthode qui permet la modélisation de deux espèces dans le même peuplement. L'observation fondamentale qui guide notre approche de combinaison des conifères et feuillus dans le même peuplement est que les arbres, peu importe l'espèce, subisse la même fermeture du couvert. Le résultat est un écartement commun peu importe l'espèce et l'écartement entre les arbres est le facteur déterminant dans les modèles de croissance. En modélisant chaque espèce selon l'espace qu'il occupe sur le peuplement, chaque espèce est modélisée comme si elle était seule sur un plus petit peuplement, et par la suite les espèces sont rassemblées pour recréer le plus grand peuplement. Cet article présente la méthode proposée et des exemples de son application à des peuplements mixtes typiques de la Nouvelle-Écosse.*

Mots clés: *modèles de croissance, multi-espèces, niveau du peuplement*

1. Introduction

In Nova Scotia, the department of Natural Resources has been publishing forest research reports since the late 1980s. All told, over 90 reports have been published on topics mostly related to modeling growth and yield and the impact of management on productivity in forest operations in the province[1]. They have developed softwood and hardwood individual stand growth and yield models. These models are based on data collected from permanent sample plots (PSP) randomly distributed around the province and the data was collected over a 40-year period at 5-year intervals. The PSPs were set up in such a way that they would represent a variety of management regimes that are called treatment types (TRT). We can divide stands and their growth models into 5 treatment types presented in table 1.

Table 1 - Treatment types

Treatment type	Description
1	Unmanaged natural stand
2	Pre-commercially thinned natural stand
3	Un-thinned plantation
4	Commercially thinned plantation
5	Commercially thinned natural stand

An unmanaged natural stand is a stand that hasn't had any silvicultural treatment applied to it since the last regeneration harvest and is growing naturally based on the land capability of where it is geographically situated. A pre-commercially thinned natural stand has been thinned to improve crop spacing and to improve growth, quality and percentage of desirable trees (Canadian Council of Forest Ministers 2010). An un-thinned plantation has only been treated enough to ensure that the stand remains a plantation at its initial planting density without any other competing vegetation. In commercial thinning, the removal of trees is delayed until the stand has enough trees of marketable value so that releasing the stand not only gives more room for the remaining trees to grow but the trees being removed can be sold to create revenue (Canadian Council of Forest Ministers 2010).

We first present growth dynamics as they are modeled in N.S. for single-species individual forest stands. We then propose an approach for combining and growing two species together in the same stand and we test the model and present results.

2. Growth dynamics

The terms growth and yield are often used together but they are two separate terms. In this paper, growth refers to the change, over time, of average stand height, average stand diameter and crown closure. Height is always a function of age and site index (SI^{θ}) where site index is the dominant tree height in meters at age 50 years. Height does not depend on TRT and is not used as a stand variable in any of the growth models. Equivalently, we could have used height and site index and treated age as a derived variable. The yield of a stand refers to the volume of wood products that are available for harvest on a stand at any given time. This paper focuses on growth only.

The data collected and analysed from PSPs has been used to create equations that define the diameter growth of HW and SW stands for each of the treatment types presented in table 1 (NSDNR (1993b), O'Keefe and McGrath (2006)). Stands of treatment types 3 and 4 are pure SW

[1] http://novascotia.ca/natr/library/publications/forestry-research.asp#research, visited November 2, 2014.

stands as there aren't any HW plantations in Nova Scotia. Stands of TRT types 1, 2 and 5 can include SW and/or HW.

In most stand level growth models, site index (SI^{θ}) is an important factor. In any given stand, there may be a site index for SW and one for HW. In this work, growing a stand means using the equations to determine the values of the stand variables 5 years in the future. In order to simplify the discussion yet show relationships between variables more clearly, the following set of variables is used in stand growth and yield modeling.

cc_t	crown closure fraction at time t
st_t	stocking (% area forested) at time t
pct_t^{θ}	percentage of total basal area of the stand at time t that is species type θ
d_t^{θ}	stand quadratic mean diameter at time t for species type θ (centimeters)
ha_t	height age for stand at time t

Crown closure is defined as a measure of the proportion of the ground covered by the vertical projection to the ground of the crown of all trees on the stand (Canadian Council of Ministers 2010). Stocking can be less than 100% if the stand has open areas such as swamps and rocky outcrops that prevent tree growth. The basal area of a tree is the round surface area of the stem of a tree when the tree is cut at 1.37m from the ground that is known as breast height (Canadian Council of Forest Ministers 2010). So a tree with a diameter at breast height of 10 cm would have a tree basal area (TBA) of 0.007854 m2 (π·r2). The total basal area of a stand is the sum of the TBA of all the trees on a stand. The quadratic mean stand diameter of the trees on a stand is a basal area weighted average. More precisely, it is the diameter of a tree of average basal area (Gove 2003). For example, if a stand has 150 m2 of basal area and 2800 trees, the stand would have a quadratic mean stand diameter of 26.12 cm ($2 \times \sqrt{(150/2800)/\pi}$). Here, quadratic mean stand diameter is referred to as diameter (d) and the total stand basal area is referred to as basal area (BA).

The t subscript in the variables above represents time. NS DNR did their data collection, at 5 year intervals, over a 40 year period. Therefore, the growth equations are built for 5 year intervals and t+5 means 5 years have elapsed.

The age discussed so far refers to the height age. It`s the actual age of the stand since it`s last regeneration harvest and is the one used to calculate the height of the dominant trees on the stand using the site index curves. Diameter age or fake age (fa_t^{θ}) is used when calculating diameter growth for certain types of managed stands. Fake age isn't the actual age of the stand but rather an age that represents the current state of the stand which may be the result of a series of silvicultural operations that modified diameter and spacing since the last regeneration harvest. With either fa_t^{θ} or ha_t, we can write $age_t + 5 = age_{t+5}$.

Once growing stock is established on the stand, growth of the stand can be calculated using the models in this section. The growth models discussed in this section can be divided into 5 sets based on the stand characteristics and they are presented in table 2. For stands of TRT types 2, 3, 4 and 5, spacing (SP_t^{θ}) and species are used to determine which set of equations to use for growth calculations. SP_t^{θ} is calculated using equation (1).

$$SP_t^{\theta} = \frac{BA_t^{\theta}}{3.1415926 \times \left(\frac{d_t^{\theta}}{200}\right)^2} \qquad (1)$$

Functions f_i presented in this section and the next are taken from NS DNR research reports and can be found in Comeau (2011).

Table 2 – Sets of forest stand growth equations

Set 1	TRT = 1 (non-species specific)
Set 2	TRT types 2, 3, 4, 5 with $SP^S \geq 3.1$m (softwood only)
Set 3	TRT types 2, 4, 5 with $SP^S < 3.1$m (softwood only)
Set 4	TRT = 3 with $SP^S < 3.1$m (softwood only)
Set 5	TRT types 2, 5 (hardwood only)

Set 1:

This set includes all unmanaged natural stands which are the easiest growth to calculate as they are always at 100% crown closure. Therefore the number of trees on the stand depends only on d_t^θ which is a function of ha_t and SI^θ. Growing the stand is simply a matter of adding 5 years to the current age (ha_{t+5}). All other stand variables are dependent on the new age ha_{t+5}. Equations (2), (3) and (4) are used to calculate d_t^θ which is required for yield calculations.

$$dht_t^\theta = f_1(ha_t, SI^\theta) \qquad (2)$$
$$aht_t^\theta = f_2(dht_t^\theta) \qquad (3)$$
$$d_t^\theta = f_3(aht_t^\theta, SI^H) \qquad (4)$$

Set 2:

This set includes softwood stands of TRT types 2, 3, 4 and 5 that have $SP_t^S \geq 3.1m$. Rather than one equation, calculating d_{t+5}^S for this set requires a sequence of preliminary values which are calculated with the equations presented in Comeau (2011). Using d_t^S and the appropriate maximum stocking line, $maxtrees_t^\theta$ is obtained. The stand crown closure, SI^S, d_t^S and $maxtrees_t^\theta$ are used to calculate d_{t+5}^S.

$$d_{t+5}^S = f_4(d_t^S, SI^S, maxtrees_t^\theta, cc_t) \qquad (5)$$

Set 3:

This set includes softwood stands of TRT types 2, 4 and 5 that have $SP_t^S < 3.1$m. We calculate a fake age (fa_t^S) corresponding to the observed spacing, diameter and site index and it may differ from the actual age of the stand (ha_t^S). For a given site index and spacing, f_5 is the inverse of f_6.

$$fa_t^S = f_5(SP_t^S, d_t^S, SI^S) \qquad (6)$$

5 years of growth are added to fa_t^S and the new diameter is calculated with fa_{t+5}^S.

$$d_{t+5}^S = f_6(fa_{t+5}^S, SP_t^S, SI^S) \qquad (7)$$

Set 4:

This set includes softwood stands of TRT type 3 that have $SP_t^S < 3.1m$. Plantations (TRT=3) are similar to natural stands in that the stand grows differently if it's been thinned but plantations don't contain any hardwood so there is only one set of growth equations. The diameter growth of a plantation with SP_t^S < 3.1m can't be calculated directly with one equation. Rather d_{t-2}^S and d_{t+3}^S are calculated and the difference between the two values is the diameter growth for the next 5 years.

$$d_{t+3}^S = f_6(ha_{t+3}^S, SP_t^S, SI^S) \quad (8)$$
$$d_{t-2}^S = f_6(ha_{t-2}^S, SP_t^S, SI^S) \quad (9)$$
$$d_{t+5}^S = d_t^S + d_{t+3}^S - d_{t-2}^S \quad (10)$$

Set 5:

This set includes all hardwood stands of TRT types 2 and 5 regardless of spacing. The hardwood diameter growth model doesn't explicitly depend on spacing therefore there is only one diameter growth model (eq. 11).

$$d_{t+5}^H = f_7(d_t^H, BA_t^H, SI^H) \quad (11)$$

The last step in the process for any single-species stand of treatment types 2, 3, 4 and 5 is to calculate crown closure (cc_{t+5}) which doesn't have a species indicator. Using d_{t+5}^θ and the appropriate maximum stocking line, $maxtrees_{t+5}^\theta$ is obtained. Using equations (12) and (13), $TFREQ_{t+5}^\theta$ and cc_{t+5} are calculated.

$$TFREQ_{t+5}^\theta = f_8(d_{t+5}^\theta) \quad (12)$$
$$cc_{t+5} = \frac{TFREQ_{t+5}^\theta}{maxtrees_{t+5}^\theta} \times 100 \quad (13)$$

The next section discusses how the growth of two species in a single stand is modeled in this work.

3. Proposed Method for Growing a Mixed-species Forest Stand

Up to this point, all growth models presented are those developed by NS DNR and they are divided into sets for easier presentation. In this section, we propose a new method for combining growth models for more than one species into a single stand mixed-species growth model. It follows directly from the models presented in the previous section and some results of applying this modelling approach to typical stands found in Nova Scotia are presented in the next section.

The previous section divided the growth models into 5 sets and, with the exception of set 1 which deals only with natural unmanaged stands, they are all single species diameter growth models. In set 1, diameter and height growth depend only on age and site index of the stand. In that case, any information related to species mix on the stand will be used solely for volume calculation and are not covered by the modelling approach proposed here. Sets 2 to 5 deal with managed natural stands and plantations and are divided according to species and/or spacing. With single species stands, the equations in the previous section can be applied directly without considering the mixed-species approach proposed here. When dealing with a mixed-species stand, the sets of growth equations in the previous section all apply as presented but we work on the assumption that each species in a multi-species stand grows as if it were alone on a smaller stand. In this section, we explain the proposed approach of how to calculate the respective spacing for each species type when they are growing together on the same stand, how to grow each of them according to their respective growth equations and how to put them back together to calculate the new values of the multi-species stand variables.

The main observation that drives our approach and the assumption above is that trees, whether softwood or hardwood, experience the same crown closure when they are on the same stand. This crown closure creates an effective spacing for each species. This then can drive all the models. In the next section, the proposed methodology is illustrated by way of some examples. The proposed approach can be summed up by the following steps:

- Step 1: Calculate the fraction of the stand covered by each species type;
- Step 2: Calculate the respective spacing for each species type. This is done based on the fact that crown closure is the same for each species type;
- Step 3: Given the spacing for each species type, calculate diameter growth using the respective growth models;
- Step 4: Combine the two species types to calculate the stand characteristics of the new stand.

What follows is a detailed explanation of each step with accompanying equations where appropriate.

Step 1: Calculate the fraction of the stand covered by each species type

The procedure starts with the calculation of the maximum stocking basal area for each species type, were each a pure single-species stand, using the function below:

$$BAFULL_t^{\theta} = maxtrees_t^{\theta} \times 3.1415926 \times \left(\frac{d_t^{\theta}}{200}\right)^2$$

Using the equations for $maxtrees_t^{\theta}$ published by NS DNR (NSDNR (1993b), O'Keefe and McGrath (2006)) which depend on d^{θ}, we determine that a pure single-species SW stand has approximately twice as much BA as a pure single-species HW stand ($BAFULL^S$ = 60m^2 vs. $BAFULL^H$ = 30m^2). With this difference of maximum BA for the two species, the fraction of the ground area of the stand covered by each species type in a mixed-species stand isn't the same as pct^S and pct^H, which are the percentages of stand total basal area for each species type. Therefore, we define $fraction^{\theta}$ as the fraction of ground area of the stand covered by species θ to yield pct^{θ} and we assume that this fraction remains unchanged until a silvicultural treatment changes it. This assumption is based on the fact that even-aged stands are defined as stands that don't have advanced regeneration that can come up underneath the canopy to fill any openings in the stand (Smith 1997). We require that

$$\sum_{\theta} fraction^{\theta} = 1$$

With two species, the solution is easy: $fraction^S$ is the solution to

$$pct^S = \frac{fraction^S \times BAFULL^S}{fraction^S \times BAFULL^S + fraction^H \times BAFULL^H}$$

where pct^S is given as a stand variable. The solution is:

$$fraction_t^S = \frac{1}{\left\{\left(\frac{1}{BAFULL_t^H}\right) \times \left[\left(\frac{BAFULL_t^S}{pct_t^S}\right) - BAFULL_t^S\right]\right\} + 1}$$

and $fraction_t^H = 1 - fraction_t^S$, which are then used to calculate spacing in step 2.

Step 2: Calculate spacing between the trees

In this step of the proposed approach, we take advantage of the fact that crown closure is the same for both species on the same stand. Given the number of trees on the stand for each species and the area of the stand covered by each species, we can calculate the spacing between SW and HW trees as they are perceived by the individual trees on the stand. It is this spacing that is used in the diameter growth models from the previous section.

If crown closure < 100%, the actual number of trees on a pure single-species stand ($TFREQ_t^\theta$) is a proportion of $maxtrees_t^\theta$ according to the following relationship:

$$TFREQ_t^\theta = maxtrees_t^\theta \times cc$$

On a mixed-species stand, a fraction of the stand is covered by each species type so we adjust the number of trees as follows:

$$TFREQ_t^\theta = fraction_t^\theta \times maxtrees_t^\theta \times cc$$

Given that, in this work, a stand is defined as being one hectare in size and that a hectare is 10,000m^2 (100m × 100m), we calculate the spacing for each species as follows:

$$SP_t^\theta = \sqrt{\frac{10000 \times fraction_t^\theta}{TFREQ_t^\theta}}$$

Given the spacing for each species type, we proceed to step 3 where diameter growth is calculated.

Step 3: Grow the species types according to their respective growth equations

What follows is an explanation of the adjustments that need to be made to each set of equations in the previous section for combining SW and HW in the same stand. Equations from set 2 are used when $SP^S \geq$ 3.1m but the growth equations don't explicitly depend on spacing. In this case, the spacing calculated above is only used to determine whether the equations from set 2 should be used. The same goes for set 5. In sets 3 and 4, the diameter growth equations explicitly depend on SP^S and SP^H. Once the spacing for each species has been calculated as described in the previous step, growth equations in sets 3 and 4 are applied as described in the previous section to determine diameter growth hence the new diameters for SW and HW. With spacing for both species types known, the appropriate set of growth equations is chosen and SW and HW diameter growth is calculated.

Step 4: Combine the species to calculate the values of the stand variables for the new stand

Given the diameter growth calculated in the previous step, we add it to d_t^θ and calculate d_{t+5}^θ which is used to calculate $maxtrees_{t+5}^\theta$ using maximum stocking lines published by NS DNR. The following equations are then used to calculate the stand characteristics post-growth.

$$TFREQ_{t+5}^\theta = fraction_t^\theta \times maxtrees_{t+5}^\theta \times \frac{cc_t}{100}$$

$$BA_{t+5}^\theta = TFREQ_{t+5}^\theta \times 3.1415926 \times \left(\frac{d_{t+5}^\theta}{200}\right)^2$$

$$pct_{t+5}^\theta = \frac{BA_{t+5}^\theta}{BA_{t+5}^S + BA_{t+5}^H}$$

$$cc_{t+5} = \frac{TFREQ_{t+5}^S}{maxtrees_{t+5}^S} \times fraction_t^S + \frac{TFREQ_{t+5}^H}{maxtrees_{t+5}^H} \times fraction_t^H$$

The reader should note that cc_t is used to calculate $TFREQ_{t+5}^\theta$. The number of trees on the stand at time $t + 5$ is a function of the number of trees that were on the stand at time t and adjusted according to the maximum number of trees that the stand can support given d_{t+5}^θ. The next section shows examples of the application of these models.

4. Mixed-species Forest Stands - Some Results from Models

The previous section presented models for growth and yield of mixed-species stands two important assumptions were made for the models to hold: in a two species stand, each species grows as if it was alone on a smaller stand, and the fraction of the ground area covered by a species doesn't change. In this section, the models are tested to show that they perform reasonably under these assumptions.

In order to show that the models developed and presented here perform reasonably, they are tested for stands with 0%, 10% and 20% HW as well as 0%, 10% and 20% SW. The results are compared with NS DNR single-species models for validation. If the models produce reasonable results for the stands listed above, this at least suggests the results will be reasonable for other stands. This is as strong a claim as can be made because models for growing mixed-species stands don't exist therefore aren't available for comparison with the proposed approach.

4.1 Comparison with Single-Species Models

Given a uniform stand, every tree in the stand sees the same crown closure (cc). But with different maximum stocking basal areas, the spacing isn't the same for each species. Furthermore, SW grows faster than HW. Therefore SW stands reach full stocking faster than HW stands when starting at the same diameter and same cc. In order for the proposed mixed-species growth model to accurately calculate the growth of each species in a mixed-species stand, the model must accurately represent SW and HW growth in pure stands. The proposed approach uses cc and species percentage to calculate the spacing between trees for each species. When a stand contains only one species, the spacing calculation is done using equation (1) and each species grows as in table 3 which shows the growth of pure SW and HW stands starting from the same d^{θ} and the same cc. The cc columns clearly show the difference in the rates at which the pure SW and HW stands reach 100% crown closure. In a mixed-species stand that combines SW and HW, the presence of SW will speed up the rate at which the HW portion of the stand reaches 100% crown closure therefore reducing its gain in diameter growth as opposed to a pure HW stand. The presence of HW in a mixed-species stand will slow the rate at which the SW portion of the stand reaches 100% crown closure therefore accelerating diameter growth of SW as opposed to a pure SW stand. In other words, adding HW to a mixed-species stand reduces total stand BA, for the same cc, which raises spacing between trees and encourages faster diameter growth. These findings are consistent with other results reported in the literature (Andreassen and Tomter (2003) and Huang and Titus (1995)). Table 3 will serve as the benchmark for comparison in order to validate the mixed-species model and all stated increases and decreases in the values of stand variables in the rest of this section are always in comparison with table 3 unless otherwise stated.

Tables 4 and 5 show the effect of 10% and 20% HW in a mixed-species stand. As expected, with the same starting d^{θ} and cc as table 3, the effect on diameter growth is exactly as described in the beginning of this section. The presence of 10% HW in a mixed-species stand gives SW a 2.16% gain in diameter growth over a 20 year period (21.25cm vs. 20.80cm). The presence of 20% HW gives SW a 2.71% gain in diameter growth over the same 20 year period (21.38cm vs. 20.80cm) when compared with the pure SW stand. The pure SW stand reaches 100% crown closure in 20 years and the presence of 10 and 20% HW only has a small effect on the rate at which the stand reaches 100% crown closure. There is not enough HW BA to have a strong influence on SW growth. These findings are consistent with the expected effect of the presence of small amounts of HW to an almost pure single-species SW stand. The smaller full stocking basal area and larger spacing of HW has the effect of creating a little more space for the SW trees to grow when the HW is present in small amounts in a predominantly SW stand.

Table 3 – Growth for pure SW and HW stands, SI^S = SI^H = 16m

$Year$	d^S	d^H	cc SW	cc HW	BA^S	BA^H
0	15.00	15.00	60	60	35.05	17.57
5	16.84	15.95	74	67	44.16	19.87
10	18.38	16.80	88	73	52.64	22.05
15	19.69	17.58	99	78	60.39	24.13
20	20.80	18.29	100	84	61.22	26.12
25	21.89	18.94	100	89	61.67	28.03
30	22.95	19.56	100	94	62.09	29.88
35	24.00	20.13	100	99	62.49	31.66
40	25.03	20.68	100	100	62.87	32.38
45	26.04	21.21	100	100	63.23	32.63

Table 4 – Growth for a mixed-species stand with 10% HW, $SI^S = SI^H$ = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	90	15.00	10	60	28.69	3.19
5	16.97	91	15.95	9	74	36.74	3.60
10	18.64	92	16.72	8	87	44.30	3.96
15	20.05	92	17.38	8	98	51.26	4.28
20	21.25	92	17.95	8	100	52.68	4.41
25	22.40	92	18.51	8	100	53.08	4.45
30	23.52	92	19.06	8	100	53.45	4.49
35	24.63	92	19.60	8	100	53.81	4.53
40	25.71	92	20.14	8	100	54.14	4.57
45	26.77	92	20.68	8	100	54.46	4.60

When we study stands that are predominantly HW with small proportions of SW, the effect on diameter growth is exactly as described in the beginning of this section. The presence of 10% SW in a mixed-species stand causes a 0.16% loss of diameter growth for HW over a 20 year period (18.26cm vs. 18.29cm). The presence of 20% SW causes a loss of 0.33% in hardwood diameter growth over the same 20 year period (18.23cm vs. 18.29cm) when compared with a pure HW stand. The pure HW stand reaches 100% crown closure in 40 years whereas the presence of 10 and 20% SW slightly reduces that time. Although the time to reach 100% crown closure is slightly affected, there is not enough BA^S present to have a strong influence on hardwood diameter growth.

Table 5 – Growth for a mixed-species stand with 20% HW, $SI^S = SI^H$ = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	80	15.00	20	60	23.39	5.85
5	16.97	82	15.95	18	73	29.95	6.61
10	18.67	83	16.74	17	84	36.22	7.28
15	20.12	84	17.41	16	95	42.10	7.88
20	21.38	84	18.00	16	100	44.81	8.40
25	22.52	84	18.56	16	100	45.15	8.48
30	23.65	84	19.11	16	100	45.47	8.56
35	24.75	84	19.65	16	100	45.76	8.63
40	25.83	84	20.19	16	100	46.05	8.71

The findings in table 6 and 7 are consistent with the expected effect of the presence of small amounts of SW to an almost pure single-species HW stand. The larger full stocking basal area and smaller spacing of SW has the effect of removing growing space for the HW trees when the SW is present in small amounts in a predominantly HW stand.

The proposed method produces reasonable results when applied to stands with small amounts of a second species in an almost pure stand and is consistent with the results in Andreassen and Tomter (2003) and Huang and Titus (1995) which at least suggests that the results will be reasonable for other stands with a higher percentage of mixed species.

Table 6 – Growth for a mixed-species stand with 10% SW, $SI^S = SI^H$ = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	10	15.00	90	60	1.85	16.65
5	16.97	11	15.95	89	67	2.37	18.82
10	18.80	12	16.80	88	74	2.90	20.87
15	20.47	13	17.56	87	80	3.45	22.82
20	22.01	14	18.26	86	86	3.98	24.67
25	23.41	15	18.90	85	91	4.50	26.44
30	24.68	15	19.50	85	96	4.97	28.14
35	25.82	15	20.06	85	100	5.19	29.44
40	26.88	15	20.60	85	100	5.22	29.68

Table 7 – Growth for a mixed-species stand with 20% SW, $SI^S = SI^H$ = 16m

Year	*Softwood*		*Hardwood*		cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H			
0	15.00	20	15.00	80	60	3.90	15.62
5	16.97	22	15.95	78	68	5.00	17.65
10	18.78	24	16.79	76	75	6.12	19.57
15	20.44	25	17.55	75	81	7.25	21.37
20	21.94	27	18.23	73	88	8.35	23.07
25	23.30	28	18.86	72	94	9.42	24.69
30	24.52	28	19.45	72	99	10.13	26.25
35	25.63	28	19.99	72	100	10.32	26.80
40	26.69	28	20.53	72	100	10.38	27.02

4.2 Mixed-species Examples

We now present three examples to demonstrate the application of the proposed model to managed and unmanaged mixed-species stands. The first example is for TRT=1 only. The focus of this example is to show the relationship between stand variables in an unmanaged natural mixed-species stand over time. The second example uses a commercially thinned natural stand (TRT=5) and demonstrates the application of the proposed methodology to managed mixed-species natural stands. The last example is used to demonstrate the difference between plantations and natural stands.

(i) Unmanaged mixed-species natural stand

The growth of unmanaged natural mixed-species stands depends only on the number of years since the last final harvest (ha_t) and the percentage of softwood naturally present in the stand (pct^S). Table 8 shows an example of a stand that starts with ha_t = 30 years and pct^S = 50%. Stand stocking is 100%. Based on the example in table 8, starting at age 30 and for rest of the life of the stand, the softwood trees always have a larger diameter than the hardwood trees but the diameters essentially gain the same amount at each time period. Based on the method proposed in the previous section, the fraction of the stand covered by each species will not change over the life of the stand, as long as the stand starts with cc = 100%, as evidenced by the results shown in table 8.

Furthermore, all stands in this paper are even-aged throughout the entire growth cycle of the stand which means that, by definition, all trees on the stand are essentially the same age. According to this definition, no new younger trees will grow in to replace the ones currently growing until after a regeneration harvest. Therefore, it makes sense for the space occupied by each species to remain unchanged when the stand is at full stocking. Because the trees grow at different rates and $maxtrees^S \neq maxtrees^H$ for a given diameter, and based on the relationship between species percentage and $fraction_t^\theta$ given in the previous section, pct^θ drifts slightly from the initial values but the change isn't significant.

Table 8 – Example of growth for a natural untreated stand with stand stocking = 100% and SI^S = SI^H = 16m at age 50 years

$Year$	Age	Softwood		Hardwood		$fraction^S$	$fraction^H$	BA^S	BA^H
		d^S	pct^S	d^H	pct^H				
0	30	7.3	50	5.9	50	0.32	0.68	14.75	14.75
5	35	8.3	49	6.9	51	0.32	0.68	15.09	15.54
10	40	9.4	49	8.0	51	0.32	0.68	15.38	16.22
15	45	10.4	48	9.0	52	0.32	0.68	15.64	16.82
20	50	11.3	48	9.9	52	0.32	0.68	15.87	17.36
25	55	12.2	47	10.8	53	0.32	0.68	16.07	17.82
30	60	13.0	47	11.6	53	0.32	0.68	16.25	18.24
35	65	13.8	47	12.4	53	0.32	0.68	16.41	18.61
40	70	14.6	47	13.1	53	0.32	0.68	16.55	18.95
45	75	15.3	46	13.8	54	0.32	0.68	16.68	19.24
50	80	15.9	46	14.4	54	0.32	0.68	16.80	19.51
55	85	16.5	46	15.0	54	0.32	0.68	16.90	19.75
60	90	17.1	46	15.5	54	0.32	0.68	16.99	19.96

(ii) Managed natural stands

An example of a commercially thinned natural stand with 60% SW basal area and 15cm diameter for SW and HW is shown in table 9. The example in table 10 has the same species split but HW has an initial diameter of 9.4 cm. There are two noticeable differences between these examples and the last example in the previous section. First, crown closure increases steadily until it reaches 100% in 25 years in table 9 and 20 years in table 10. The fraction of a stand covered by each species changes until the stand reaches crown closure at which point it stops changing. pct^θ changes only slightly as it did in the previous example. Second, the diameter is the same for SW and HW in table 9 which could not occur in an unmanaged natural stand. Because these stands may have been treated since the last final harvest, the diameter growth could have been altered from its natural state.

In table 9, SW and HW have the same initial diameter. The result is a rapid change in pct^θ until crown closure is reached which is caused by the faster SW diameter growth and larger number of SW trees on the stand. In table 10, there are more hardwood trees because of the smaller d^H. This results in a higher $fraction^H$ and a higher $TFREQ^H$ which allows the HW to retain its fraction of the stand area. The larger $TFREQ^H$ allows the HW to gain basal area faster in table 10 compared to table 9.

Table 9 – Example of growth for a stand with TRT = 5 and $SI^S = SI^H$ = 16m at age 50 years with identical starting diameters

Year	*Softwood*		*Hardwood*		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H					
0	15.0	60	15.0	40	0.429	0.571	60	15.05	10.03
5	16.8	63	15.9	37	0.457	0.543	70	18.96	11.34
10	18.5	65	16.8	35	0.479	0.521	79	22.82	12.53
15	19.9	66	17.5	34	0.496	0.504	88	26.55	13.62
20	21.2	67	18.1	33	0.510	0.490	96	30.09	14.63
25	22.3	67	18.7	33	0.510	0.490	100	31.55	15.37
30	23.4	67	19.2	33	0.510	0.490	100	31.76	15.51
35	24.4	67	19.8	33	0.510	0.490	100	31.96	15.65

Table 10 – Example of growth for a stand with TRT = 5 and $SI^S = SI^H$ = 16m at age 50 years with different starting diameters

Year	*Softwood*		*Hardwood*		$fraction^S$	$fraction^H$	cc	BA^S	BA^H
	d^S	pct^S	d^H	pct^H					
0	15.0	60	9.4	40	0.394	0.606	60	13.81	9.21
5	16.8	60	10.5	40	0.402	0.598	73	17.40	11.41
10	18.4	61	11.3	39	0.409	0.591	85	20.80	13.42
15	19.8	61	12.1	39	0.415	0.585	95	23.98	15.26
20	20.9	61	12.8	39	0.415	0.585	100	25.41	16.30
25	22.0	61	13.4	39	0.415	0.585	100	25.59	16.55
30	23.1	61	14.0	39	0.415	0.585	100	25.77	16.78
35	24.1	60	14.6	40	0.415	0.585	100	25.93	17.00

(iii) Comparison of plantations and natural stands

In forests that are managed for maximum economic gain, forest managers often opt for plantations over natural stands when choosing what type of stand will replace another after a regeneration harvest. The last example in this section isn't used to show the progression of a stand over time but rather to show the difference between natural stands and plantations which may serve to illustrate why many jurisdictions around the world are seeing natural forests being replaced by plantations when the main management objective is of a purely economic nature. Tables 11 and 12 show the contrast between natural and plantation stands. These values are taken directly from the NS DNR growth and yield models.

Table 11 – Example of an unmanaged natural stand (TRT = 1), $SI^S = SI^H$ = 16m at age 50 years, *ha* = 70 years

Softwood		*Hardwood*		$fraction^S$	$fraction^H$	BA^S	BA^H
d^S	pct^S	d^H	pct^H				
15.0	100	0	0	1.000	0.000	51.12	0

Table 12 – Example of an non-commercially thinned plantation (TRT = 3), $SI^S = SI^H$ = 16m at age 50 years, *ha* = 70 years

Softwood		*Hardwood*		$fraction^S$	$fraction^H$	BA^S	BA^H
d^S	pct^S	d^H	pct^H				
15.0	100	0	0	1.000	0.000	58.30	0

Plantations don't contain hardwoods in N.S. therefore the comparison is done with a single-species SW natural stand. The main difference is the amount of BA in a plantation compared to a natural stand. The trees in plantations are planted in a very organized pattern with very little wasted space which allows the plantation to have a slightly higher number of trees per hectare at full stocking when compared to natural stands. Essentially, for the same diameter, $maxtrees^S$ for a plantation is higher than $maxtrees^S$ for a natural stand which yields a higher BA for a plantation.

5. Conclusion

Development of this mixed-species model is based on the idea that all trees on a stand, regardless of species type, experience crown closure as being the same. The examples presented in this paper rely on the fact that crown closure is directly related to spacing between the trees on a stand and, in the growth and yield models published by NS DNR, spacing is the major driving force of individual SW and HW diameter growth models. The models and examples in this paper have led to an important principle: slow growing species with a low maximum density such as hardwood, when mixed with faster growing higher density species such as softwood, will give the faster growing species more room to grow by slowing the rate at which the stand reaches crown closure thus resulting in a gain in diameter growth. The proposed growth model can be used in forest management models where the objective is to optimize the management of mixed-species stands.

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