Phase Conjugation of Orbital Angular Momentum in Cylindrical Vector Beams by SBS

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CLEO 2022 – Brillouin Process
20 May 2022

Cylindrical Vector Beams (CVBs)

$$\vec{u} = \exp(i\alpha)\sin(\theta/2)\exp(-i\varphi)\hat{e}_L + \cos(\theta/2)\exp(i\varphi)\hat{e}_R$$

PRL **112,** 215001 (2014) PHYSICAL REVIEW LETTERS week ending 30 MAY 2014

Nonlinear Laser Driven Donut Wakefields for Positron and Electron Acceleration

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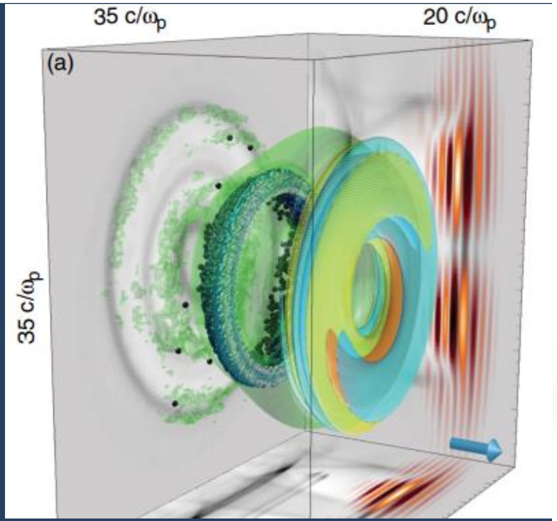
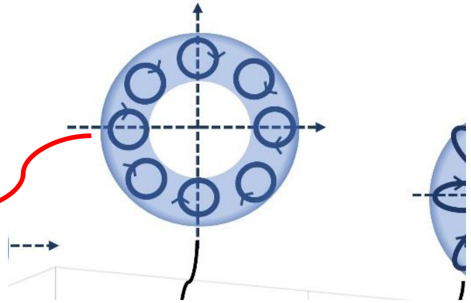
(Received 5 February 2014; published 27 May 2014)

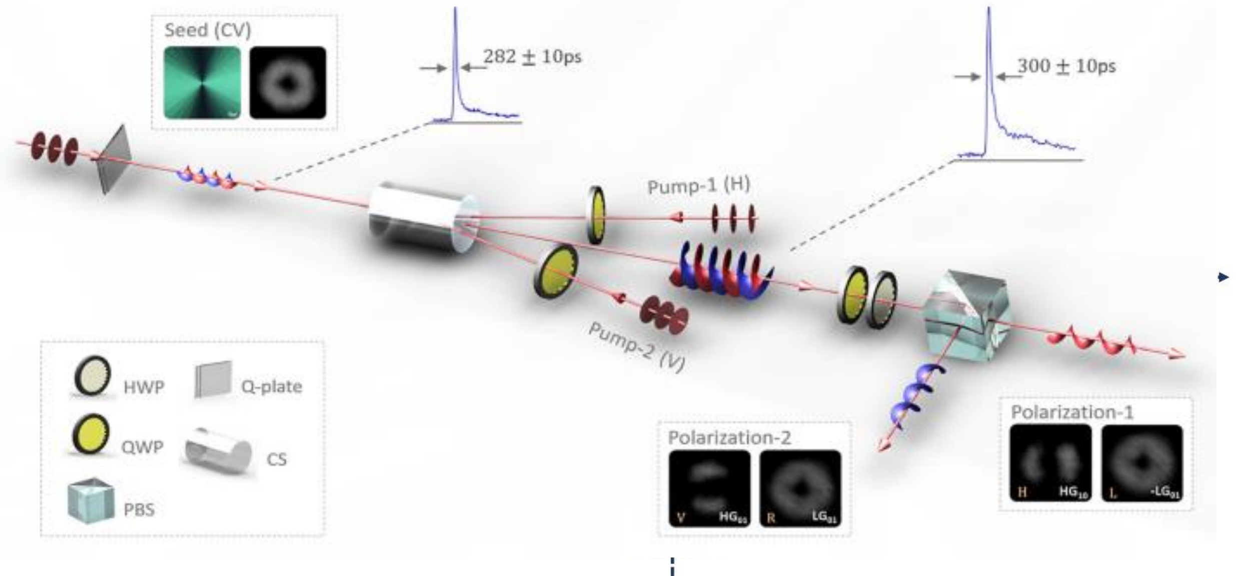




APPLIED PHYSICS LETTERS **110**, 141104 (2017)

Generation of strong cylindrical vector pulses via stimulated Brillouin amplification

Zhi-Han Zhu,[1,a),b)] Peng Chen,[2,a)] Li-Wen Sheng,[3] Yu-Lei Wang,[3] Wei Hu,[2,c)] Yan-Qing Lu,[2] and Wei Gao[1]



Appl. Sci. **2013**, *3*, 70-93; doi:10.3390/app3010070

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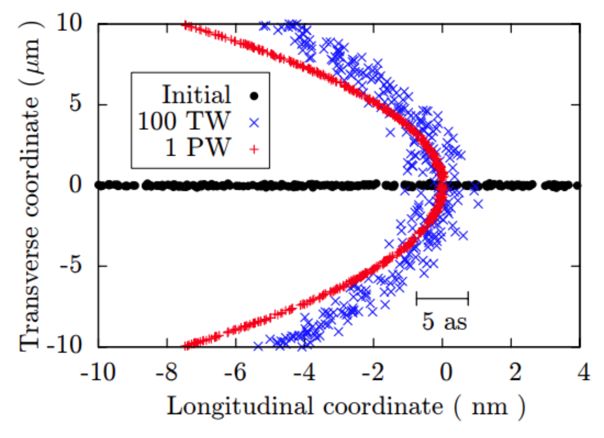
applied sciences
ISSN 2076-3417
www.mdpi.com/journal/applsci

Review

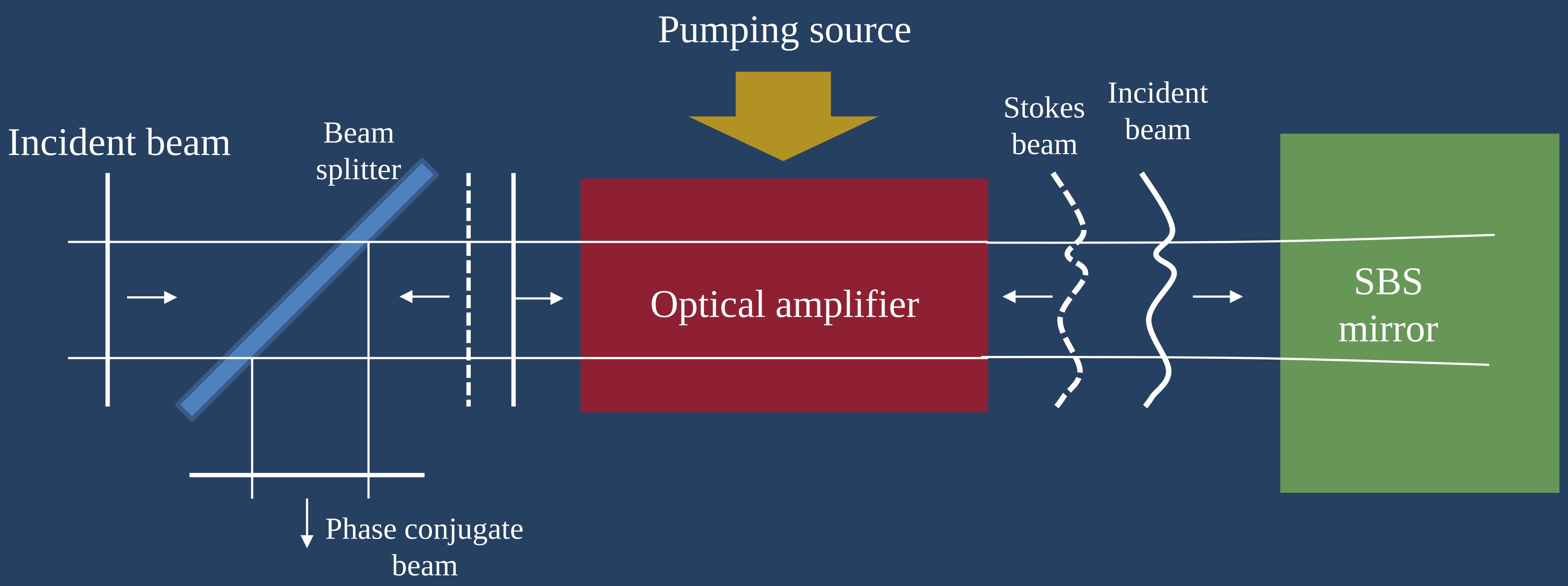
Direct Electron Acceleration with Radially Polarized Laser Beams

Charles Varin [1,*], Stéphane Payeur [2,*], Vincent Marceau [3], Sylvain Fourmaux [2], ...idt [2], Pierre-Louis Fortin [3], Nicolas Thiré [2], Thomas Brabec [1], ...e Kieffer [2] and Michel Piché [3,*]



Stimulated Brillouin Scattering (SBS) in Free Space

- Used for **phase conjugation** of wavefront distorsions in high-energy lasers



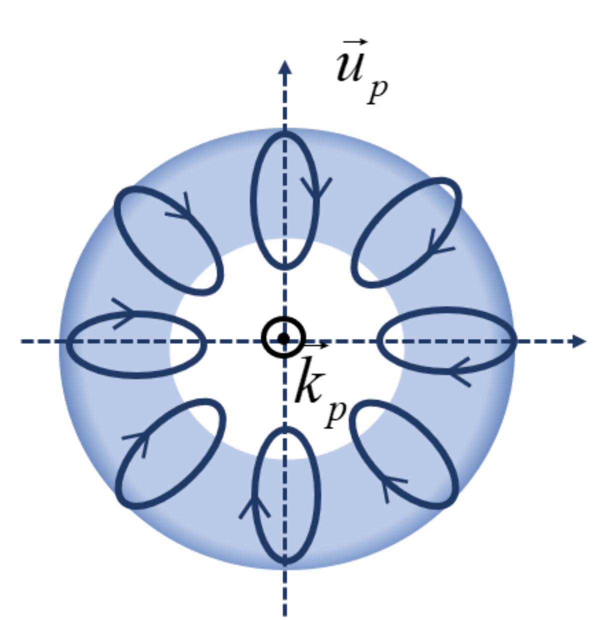
- The pump and Stokes beams have the same polarization (**scalar process**)

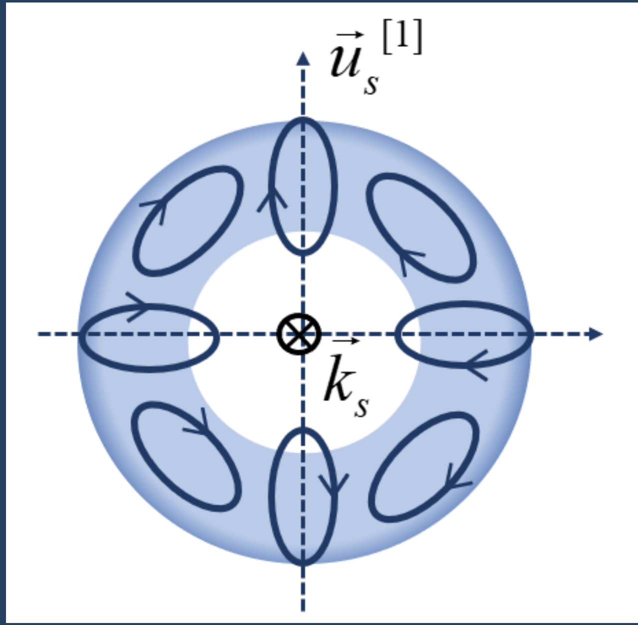
Conflicting Aspects of **Scalar Phase Conjugation** With CVBs

In CVBs, the phase and the polarization states are entangled quantities.

SBS

$$\vec{u}_p = \sin(\theta/2)\exp(-i\varphi)\hat{e}_L + \cos(\theta/2)\exp(i\varphi)\hat{e}_R \quad \Longrightarrow \quad \vec{u}_s^{[1]} = \sin(\theta/2)\exp(i\varphi)\hat{e}_L + \cos(\theta/2)\exp(-i\varphi)\hat{e}_R$$





Conflicting Aspects of Scalar PC With CVBs

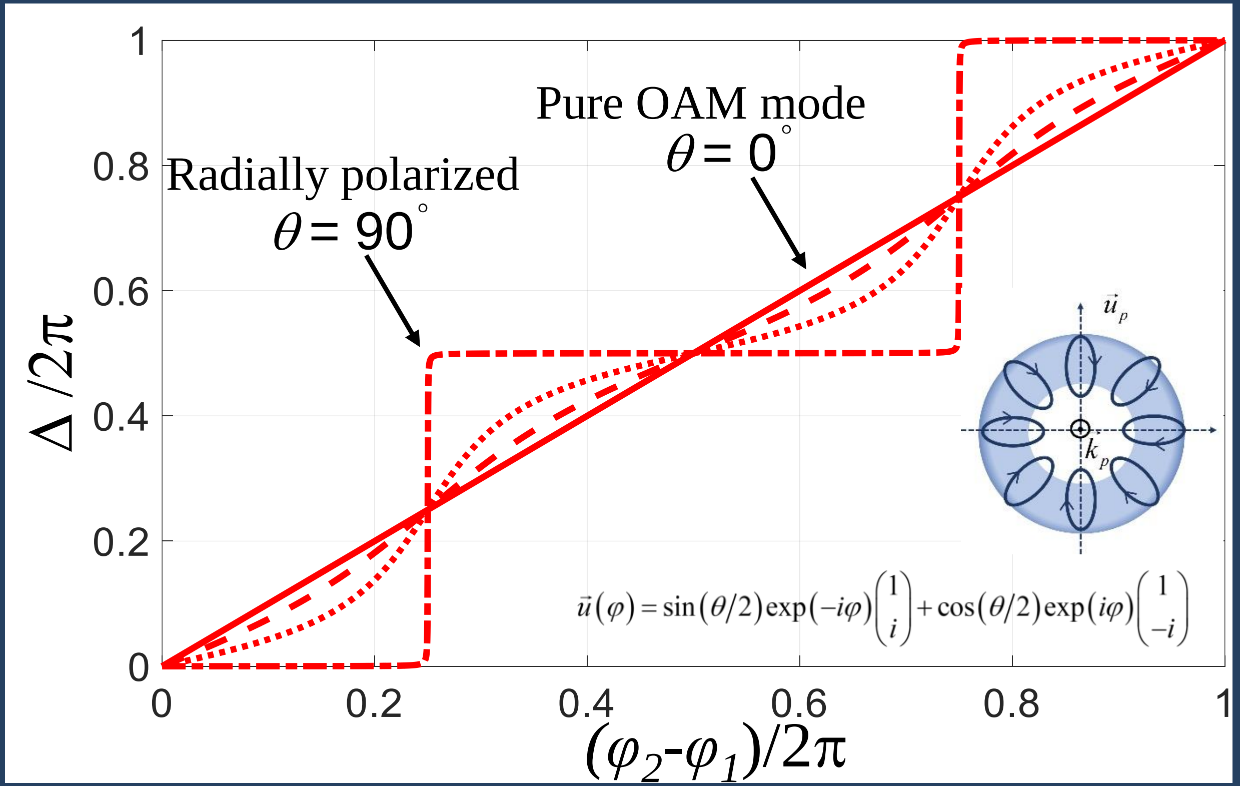
- The changing polarization state with φ entails a contribution to the phase that adds up to the local phase variations.
- This is called the geometrical phase

$$\vec{u}_p = \sin(\theta/2)\exp(-i\varphi)\hat{e}_L + \cos(\theta/2)\exp(i\varphi)\hat{e}_R$$

$$\delta\Delta = \angle\{\langle u(\varphi_1)|u(\varphi_1+\delta\varphi)\rangle\} \simeq \delta\varphi\cos(\theta)$$

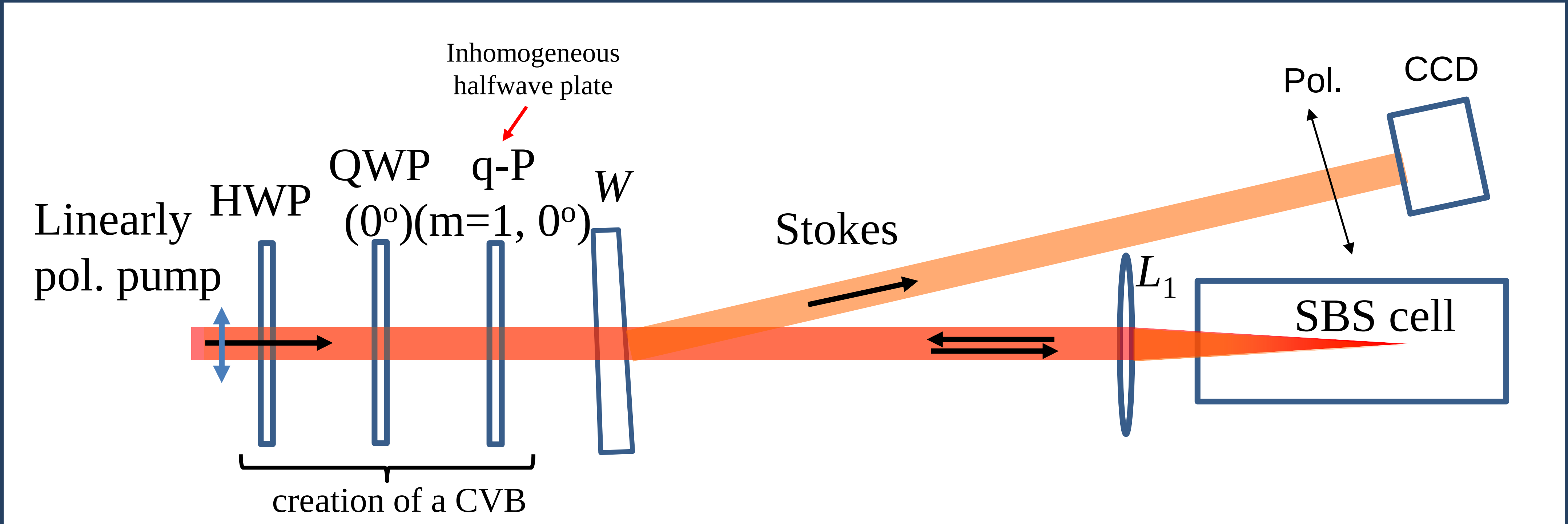
$$\Delta = \angle\{\langle u(\varphi_1)|u(\varphi_2)\rangle\} = \arctan(\tan(\varphi_2-\varphi_1)\cos(\theta))$$

$$\neq \int_{\varphi_1}^{\varphi_2} \delta\Delta$$



What happens when a CVB is reflected by a SBS mirror?

Experimental Setup

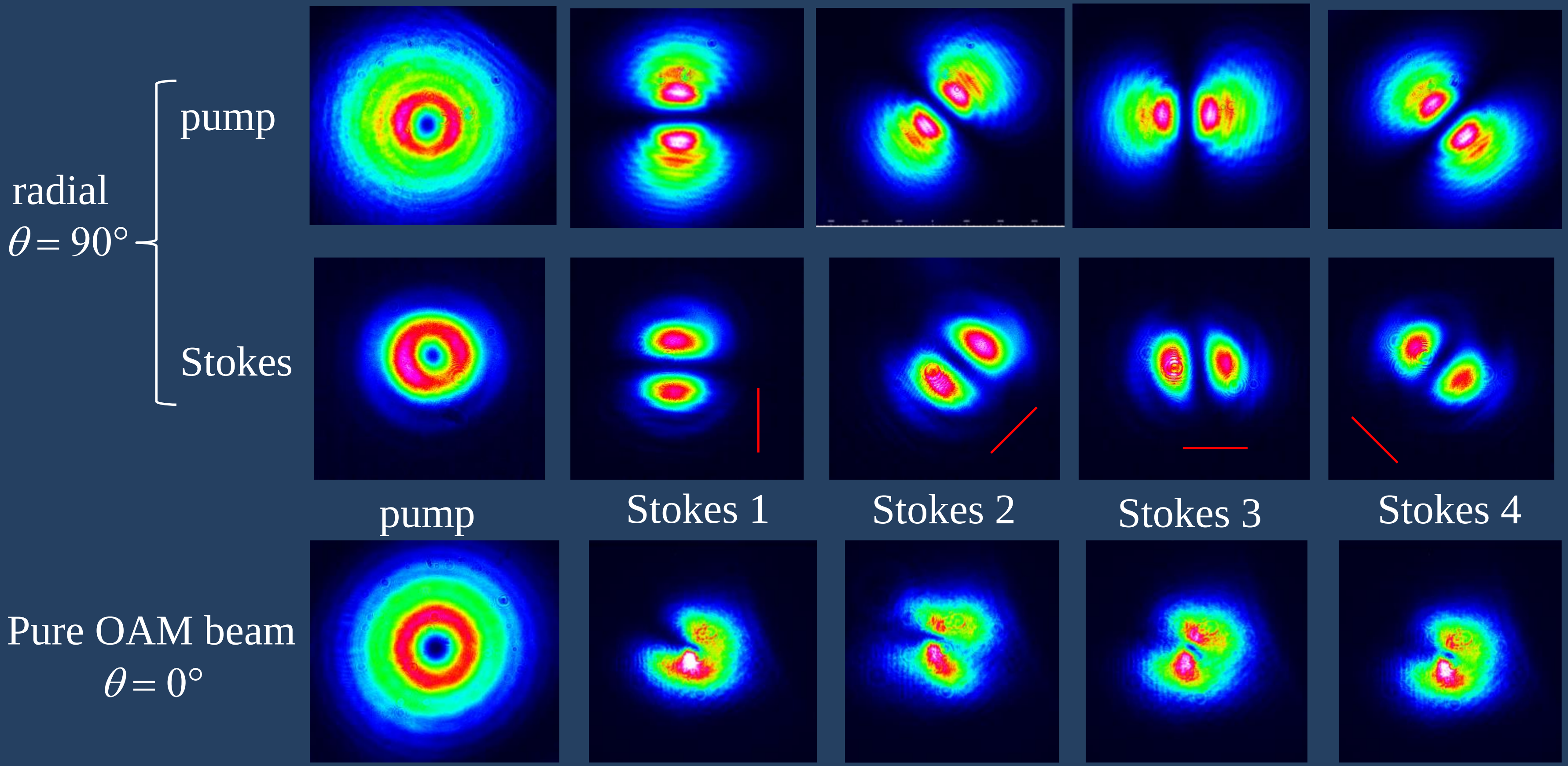


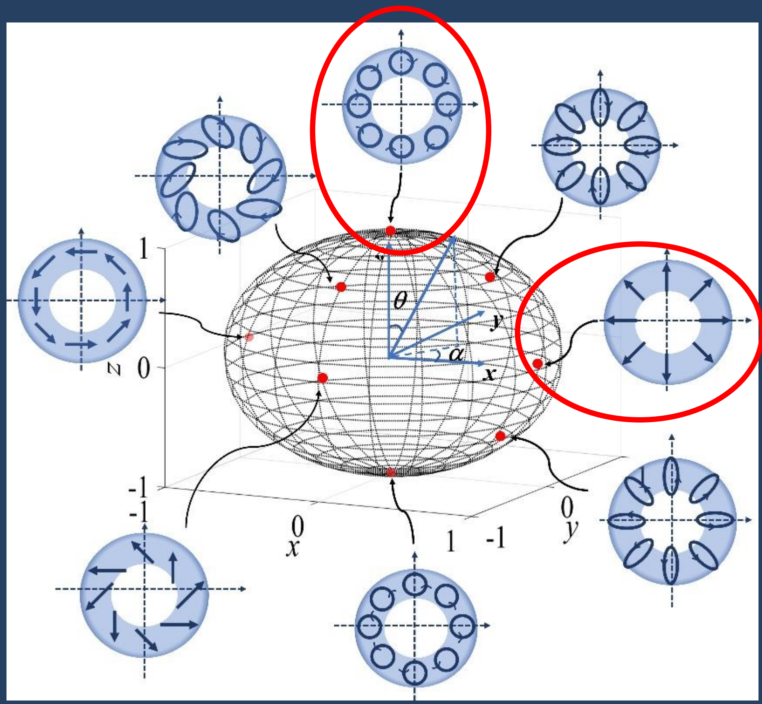
Laser source: single longitudinal, single transverse mode, horizontally polarized Q-switched Nd:YAG laser (λ=1064 nm)

Pulse energy: 3.5 mJ ; Pulse width: 3 ns

SBS medium: CCl_4 ; threshold of SBS: 1 mJ

Intensity Profile and Polarization Distribution



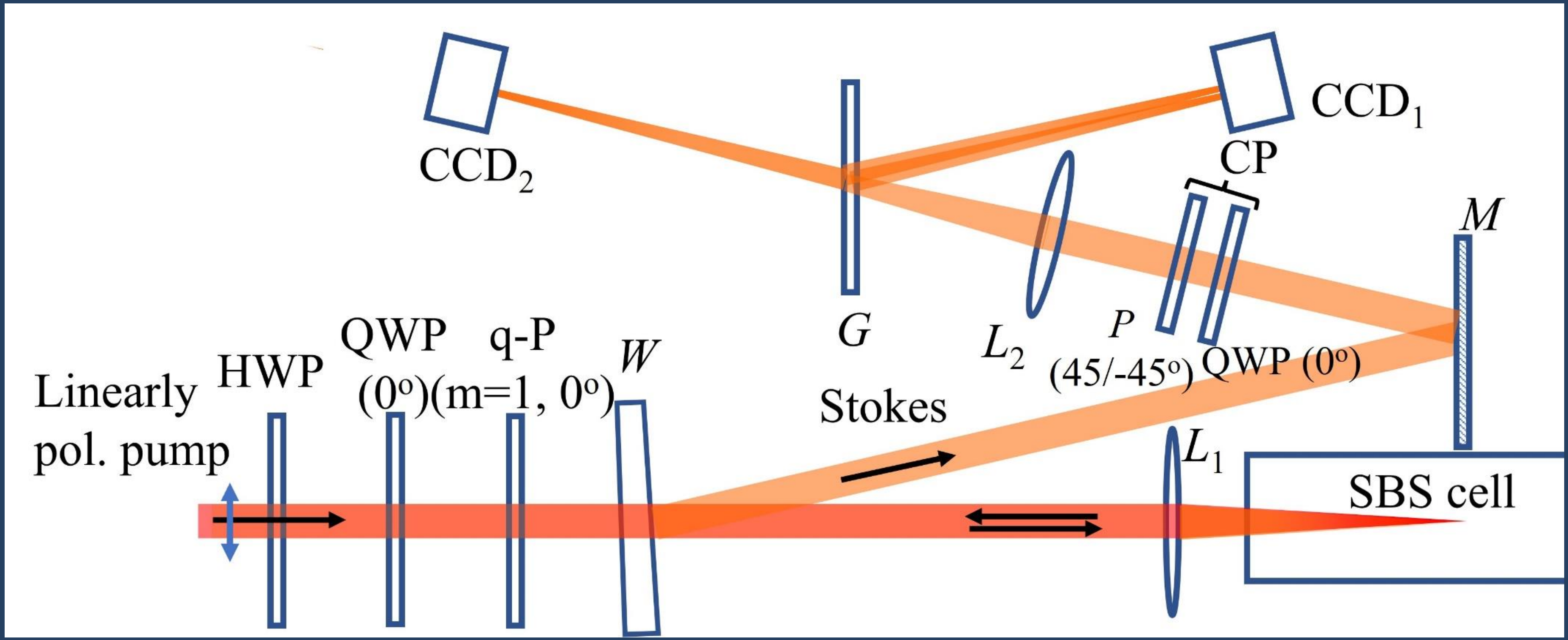


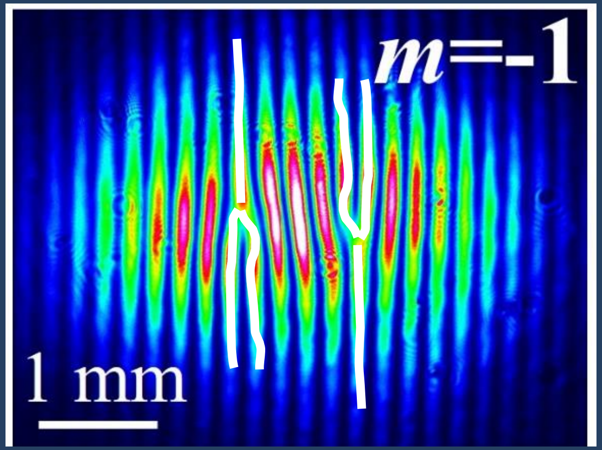
Fidelity of the intensity pattern:

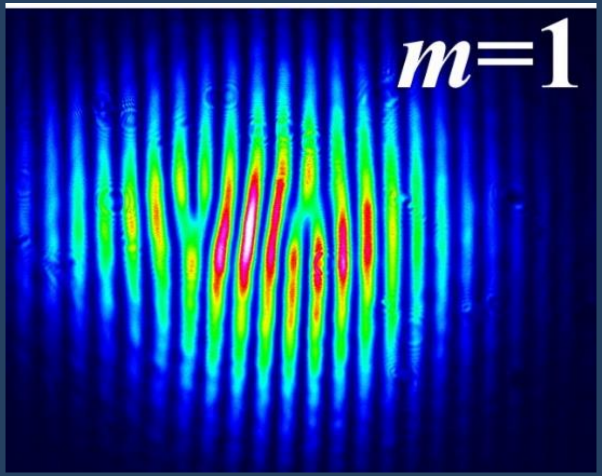
- Radially polarized : good
- Pure OAM : poor

Measurement of the OAM

$$\vec{u}_s = \exp(im\varphi)\hat{e}_R$$







- OAM obtained by shearing interferometry:
  - Each circularly polarized component is separately analyzed.
  - OAM is confirmed by the presence of two dislocations of opposite orientations in the fringe pattern.
  - Each condition is repeated about 50 times for statistical analysis.

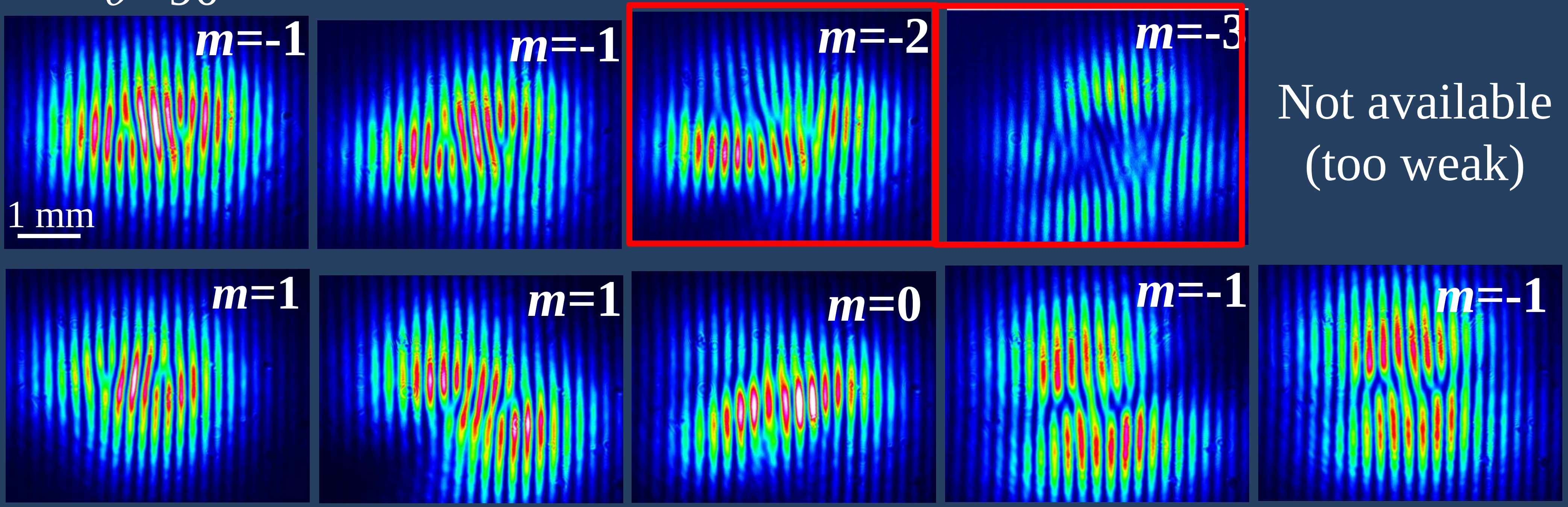
Interferograms of Stokes Beams

$$\vec{u}_p = \sin(\theta/2)\exp(-i\varphi)\hat{e}_L + \cos(\theta/2)\exp(i\varphi)\hat{e}_R$$

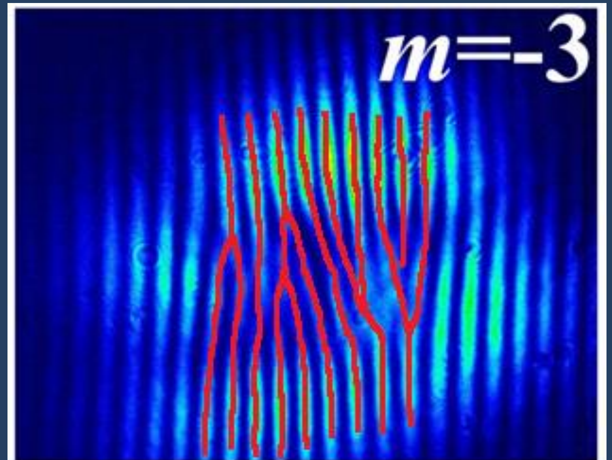
$\theta = 90^\circ$ | 74° | 58° | 26° | 2°

Left circular

right circular



Not available (too weak)



- A gradual shift of the OAM values is noticed from the equator towards the pole of the HOPS:

  (m=-1, **m=1**)$_{\text{left, right}}$ → (m=-2, **m=0**) → (m=-3, **m=-1**)

  (radial) ⟶ (pure OAM)

- No phase conjugation of OAM takes place, except near the poles.

The Theoretical Model

- We use a **vectorial** SBS theory in the paraxial approximation and with a **modal decomposition**.
- The focus is on the topological charges of the OAM modes.

Details can be found in:

J.-F. Bisson, Phys. Rev. A, 105, 053501 (2022), http://arxiv.org/abs/2205.06290

Theory : the Stokes Eigenmode Equation

- We obtain a differential matrix equation that enables the calculation of the structure of Stokes eigenmodes and eigenvalues.

$$\vec{E}_p \sim \left(\sin(\theta/2)\exp(-i\varphi) \;\; \cos(\theta/2)\exp(i\varphi)\right)_{\{\hat{e}_L,\hat{e}_R\}} \Longrightarrow \vec{E}_s \sim \left(b_{n-2}\exp\left(i(n-2)\varphi\right) \;\; \mathrm{b}'_n\exp\left(in\varphi\right)\right)_{\{\hat{e}_L,\hat{e}_R\}}$$

$$\frac{\partial}{\partial z}\begin{pmatrix} b_{n-2} \\ b'_n \end{pmatrix} = M\begin{pmatrix} b_{n-2} \\ b'_n \end{pmatrix} \quad \text{where} \quad M = \begin{pmatrix} \sin^2\left(\frac{\theta}{2}\right)\xi_{11} & \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\exp(-i\alpha)\xi_{12} \\ \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\exp(i\alpha)\xi_{21} & \cos^2\left(\frac{\theta}{2}\right)\xi_{22} \end{pmatrix}$$

ξ_{ij} : overlap integral of pump and Stokes modes

- The combination of OAM modes with topological charges (n-2) and n is the only one that keeps the cylindrical symmetry of the beam.

Theory: the Gain of the Stokes Modes

$$\vec{E}_p \sim \big(\sin(\theta/2)\exp(-i\varphi) \;\; \cos(\theta/2)\exp(i\varphi)\big)_{\{\hat{e}_L,\hat{e}_R\}} \Longrightarrow \vec{E}_s \sim \big(b_{n-2}\exp(i(n-2)\varphi) \;\; b'_n\exp(in\varphi)\big)_{\{\hat{e}_L,\hat{e}_R\}}$$

Relative gain
Axisymmetrical pump

θ (deg.)	$(n-2, n) = (-4, -2)$	$(-3, -1)$	$(-2, 0)$	$(-1, 1)$	$(0, 2)$
90	0.51	0.71	0.80	**1.00**	0.80
82	0.54	0.74	0.82	**1.00**	0.78
74	0.57	0.78	0.84	**1.00**	0.77
66	0.60	0.82	0.87	**1.00**	0.76
58	0.63	0.86	0.89	**1.00**	0.75
50	0.66	0.89	0.92	**1.00**	0.75
42	0.69	0.92	0.94	**1.00**	0.74
34	0.71	0.95	0.96	**1.00**	0.75
26	0.72	0.97	0.98	**1.00**	0.75
18	0.74	0.98	0.99	**1.00**	0.75
10	0.75	**1.00**	**1.00**	**1.00**	0.75
2	0.75	**1.00**	**1.00**	**1.00**	0.75

Radial polarization

Pure scalar OAM mode (m=1)

conjugation of the stronger OAM component

Gaussian TEM_{00}

No conjugation of OAM

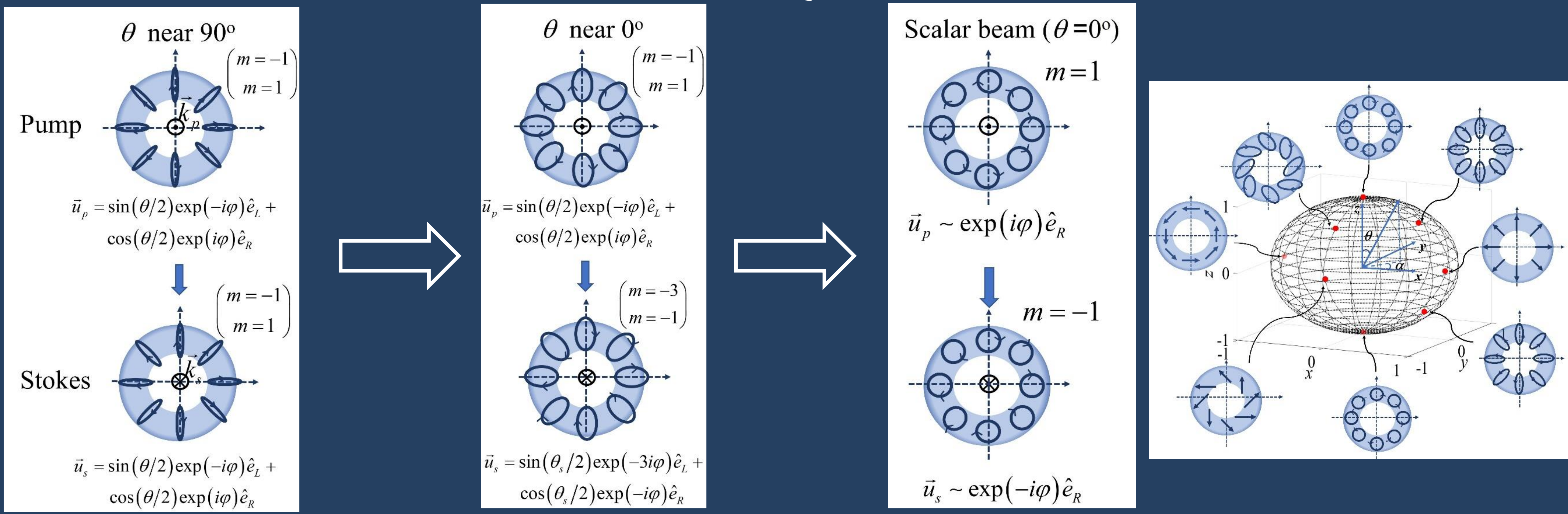
Relative gain
Asymmetric pump

$$A_{m,\varepsilon} = LG_m + \varepsilon\,\mathrm{sign}(m)LG_{-m}$$

θ (deg.)	$(n-2, n) = (-4, -2)$	$(-3, -1)$	$(-2, 0)$	$(-1, 1)$	$(0, 2)$
90	0.48	0.68	0.75	**0.92**	0.75
82	0.51	0.72	0.77	**0.92**	0.73
74	0.54	0.76	0.79	**0.92**	0.72
66	0.57	0.80	0.82	**0.92**	0.71
58	0.60	0.84	0.84	**0.92**	0.71
50	0.63	0.88	0.87	**0.92**	0.71
42	0.66	0.91	0.89	**0.92**	0.71
34	0.68	**0.94**	0.92	0.92	0.71
26	0.69	**0.97**	0.93	0.92	0.71
18	0.71	**0.98**	0.95	0.92	0.72
10	0.72	**1.00**	0.95	0.92	0.72
2	0.72	**1.00**	0.96	0.92	0.72

conjugation of the stronger m=1 OAM component

Summary



- The phase conjugation of OAM of CVBs with SBS is absent in most of the HOPS.
- Near the poles of the HOPS:
  - the CVB acquires a global helical phase factor exp(-2iφ) that keeps the cylindrical nature of the beam.
  - the stronger OAM component of the Stokes beam acquires the conjugate topological charge.
- A vectorial theory using a modal decomposition reached similar conclusions.

Conclusion

The geometrical phase inhibits the phase conjugation of the topological charges of CVBs in SBS, except at the limit of pure OAM beams.

Thank you for your attention!

Acknowledgments

Yves Christian Nonguierma
Pierre St-Onge

PhD positions are available!

Contact: jean-francois.bisson@umoncton.ca

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The Theoretical Model

[J.-F. Bisson, Phys. Rev. A, 105, 053501 (2022)]

- We use a **vectorial** SBS theory supplemented by a modal decomposition analysis.
- The focus is on PC of superpositions of OAM modes.

$$\nabla^2 \vec{E} - \frac{\kappa}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \vec{P}^{NL}}{\partial t^2}$$

$$\vec{E} = \vec{E}_p(\vec{r})\exp\left(-i\left(kz+\omega_p t\right)\right) + \vec{E}_s(\vec{r})\exp\left(i\left(kz-\omega_s t\right)\right)$$

Paraxial approx.

$$\nabla_\perp^2 \vec{E}_s + 2ik\frac{\partial \vec{E}_s}{\partial z} = -\frac{\omega^2}{\varepsilon_0 c^2}\vec{P}^{NL}_{ikz}$$

$$\cancel{P^{NL}_{+ikz} = -\varepsilon_0 \chi_{SBS}^{(3)}\left(E_p^{*} \cdot E_s\right)E_p}$$

Vectorial theory

$$\vec{P}^{NL}_{ikz} = -\varepsilon_0 \chi_{SBS}^{(3)}\left(\vec{E}_p^{*} \cdot \vec{E}_s\right)\vec{E}_p$$

Modal decomposition:

$$\vec{E}_p = \left(\sum_j a_j(z) A_j(r,\varphi,z) \quad \sum_j a'_j(z) A_j(r,\varphi,z)\right)^T \qquad \vec{E}_s = \left(\sum_j b_j(z) B_j(r,\varphi,z) \quad \sum_j b'_j(z) B_j(r,\varphi,z)\right)^T$$

$$A_{mp}(r,\varphi,z) = B_{mp}(r,\varphi,z) = C_{mp}(r,z)\exp\left(im\varphi\right)$$

The Theoretical Model (continued)

$$\left[\begin{aligned}\vec{E}_p &= \left(\sum_j a_j(z) A_j(r,\varphi,z) \quad \sum_j a'_j(z) A_j(r,\varphi,z)\right)^T \\ \vec{E}_s &= \left(\sum_j b_j(z) B_j(r,\varphi,z) \quad \sum_j b'_j(z) B_j(r,\varphi,z)\right)^T\end{aligned}\right] \Rightarrow \left[\begin{aligned}\nabla_\perp{}^2 \vec{E}_s + 2ik\frac{\partial \vec{E}_s}{\partial z} &= -\frac{\omega^2}{\varepsilon_0 c^2}\vec{P}^{NL}_{ikz} \\ \vec{P}^{NL}_{ikz} &= -\varepsilon_0 \chi^{(3)}_{SBS}\left(\vec{E}_p{}^* \cdot \vec{E}_s\right)\vec{E}_p\end{aligned}\right] \Rightarrow \begin{pmatrix}\sum_j B_j \frac{\partial b_\alpha}{\partial z} \\ \sum_j B_j \frac{\partial b'_\alpha}{\partial z}\end{pmatrix} = g\sum_{j,l,m}\begin{pmatrix}\left(b_l a^*_j a_m + b'_l a'^*_j a_m\right) B_l A_j{}^* A_m \\ \left(b_l a^*_j a'_m + b'_l a'^*_j a'_m\right) B_l A_j{}^* A_m\end{pmatrix}$$

$$\int B_n^* \begin{pmatrix}\sum_j B_j \frac{\partial b_\alpha}{\partial z} \\ \sum_j B_j \frac{\partial b'_\alpha}{\partial z}\end{pmatrix} \mathrm{d}^2\vec{r} = \int B_n^* \, g\sum_{j,l,m}\begin{pmatrix}\left(b_l a^*_j a_m + b'_l a'^*_j a_m\right) B_l A_j{}^* A_m \\ \left(b_l a^*_j a'_m + b'_l a'^*_j a'_m\right) B_l A_j{}^* A_m\end{pmatrix} \mathrm{d}^2\vec{r} \Rightarrow \left\{\begin{pmatrix}\frac{\partial b_n}{\partial z} \\ \frac{\partial b'_n}{\partial z}\end{pmatrix} = g\sum_{j,l,m}\begin{pmatrix}b_l a^*_j a_m + b'_l a'^*_j a_m \\ b_l a^*_j a'_m + b'_l a'^*_j a'_m\end{pmatrix}\xi_{s,jlmn}\right.$$

Overlap integral:

$$\xi_{s,jlmn}(z) = \int B_l A_j{}^* A_m B_n{}^* d^2\vec{r}$$

Undepleted pump approx.: $\left[\begin{aligned}&\vec{E}_p \sim \begin{pmatrix}a_{-1}A_{-1} \\ a'_1 A_1\end{pmatrix}_{\{\hat{e}_L,\hat{e}_R\}} \\ &a_{-1} = \sin(\theta/2)\exp(-i\alpha) \\ &a'_1 = \cos(\theta/2)\end{aligned}\right]$

Selection rule: $\xi_{jlmn} = 0 \quad \text{if } j - l - m + n \neq 0$

$$\Rightarrow \left\{\begin{aligned}&\frac{\partial}{\partial z}\begin{pmatrix}b_{n-2} \\ b'_n\end{pmatrix} = M\begin{pmatrix}b_{n-2} \\ b'_n\end{pmatrix} \\ &M = \begin{pmatrix}\sin^2\left(\frac{\theta}{2}\right)\xi_{11} & \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\exp(-i\alpha)\xi_{12} \\ \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\exp(i\alpha)\xi_{21} & \cos^2\left(\frac{\theta}{2}\right)\xi_{22}\end{pmatrix} \\ &\xi_{11} \equiv \xi_{-1,n-2,-1,n-2} \quad \xi_{12} \equiv \xi_{1,n,-1,n-2} \quad \xi_{21} \equiv \xi_{-1,n-2,1,n} \quad \xi_{22} \equiv \xi_{1,n,1,n}\end{aligned}\right.$$

Theory : the Stokes eigenmode equation

$$\frac{\partial}{\partial z}\begin{pmatrix} b_{n-2} \\ b'_n \end{pmatrix} = M \begin{pmatrix} b_{n-2} \\ b'_n \end{pmatrix} \quad \text{where} \quad M = \begin{pmatrix} \sin^2\left(\frac{\theta}{2}\right)\xi_{11} & \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\exp(-i\alpha)\xi_{12} \\ \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\exp(i\alpha)\xi_{21} & \cos^2\left(\frac{\theta}{2}\right)\xi_{22} \end{pmatrix}$$

Overlap integral:

$$\xi_{11} \equiv \xi_{-1,n-2,-1,n-2}$$
$$\xi_{12} \equiv \xi_{1,n,-1,n-2}$$
$$\xi_{21} \equiv \xi_{-1,n-2,1,n}$$
$$\xi_{22} \equiv \xi_{1,n,1,n}$$

Key points:

1. The Stokes eigenmodes have the form: $\vec{E}_s = \left(b_{n-2}\exp\left(i(n-2)\varphi\right) \quad b_n \exp\left(in\varphi\right)\right)^T_{\{\hat{e}_L,\hat{e}_R\}}$

   a. The values of b_{n-2} and b_n correspond to the components of the eigenvectors of matrix *M*.

   b. The corresponding eigenvalue indicates the gain of each mode.

2. Matrix *M* is Hermitian => eigenvectors are orthogonal and eigenvalues are real
3. Each vector mode behaves independently in this model
4. The combination of modes with topological charges (*n*-2) and *n* is the only one that keeps the cylindrical symmetry of the Stokes beam identical to the pump beam.