Exceptional Points in the Polarization Space with Anisotropic Materials

Jean-François Bisson

Department of Physics and Astronomy

Université de Moncton

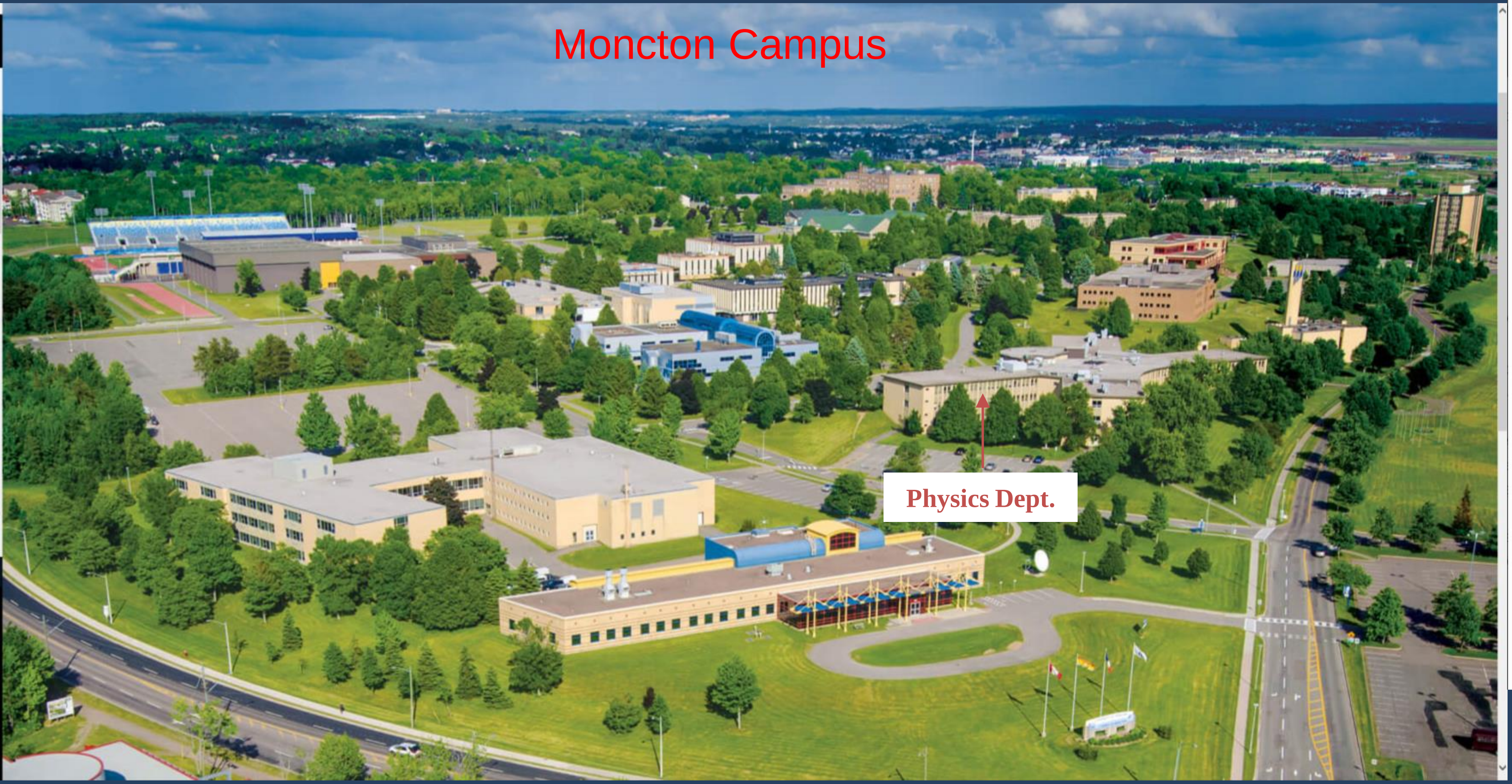
Seminar at Michigan Technological University

31 March 2022

UNIVERSITÉ DE MONCTON
CAMPUS DE MONCTON

Université de Moncton

- French speaking university created in 1963
- Around 5 000 students
  [Moncton (main campus), Shippagan, Edmundston]



Hopewell Rocks, NB, Canada

Low tide

High tide

Thin films and Photonics Research Group

- Founded in 1984
- About 20 members
- Collaborations with academic and private sectors

Research fields

- Nonlinear optics
- Thin films and photonic crystals
  - Organic Light Emitting Diodes (OLED) and photovoltaic systems
  - Chromogenic materials (photochromic, thermochromic, electrochromic)
  - Pulsed laser deposition
  - Measurement techniques (spectrophotometry, ellipsometry, AFM, etc)
- Laser physics

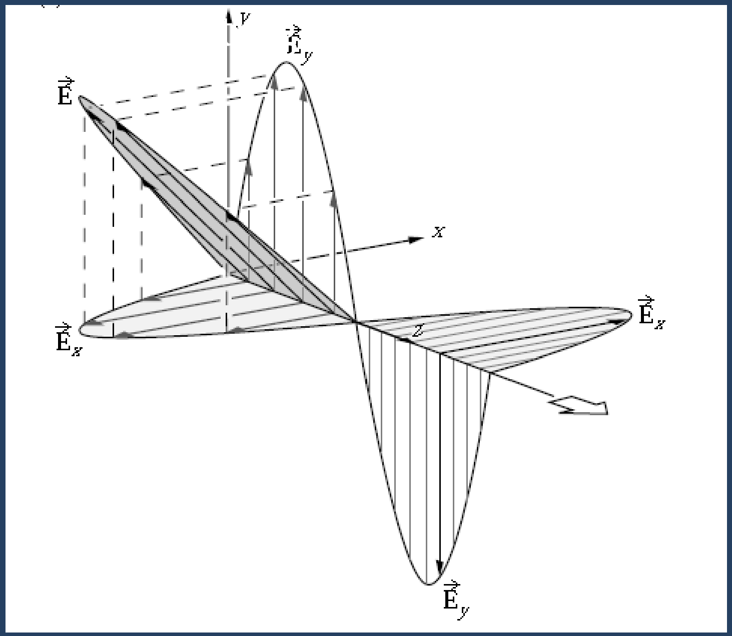
PhD positions are available!

Contact: jean-francois.bisson@umoncton.ca

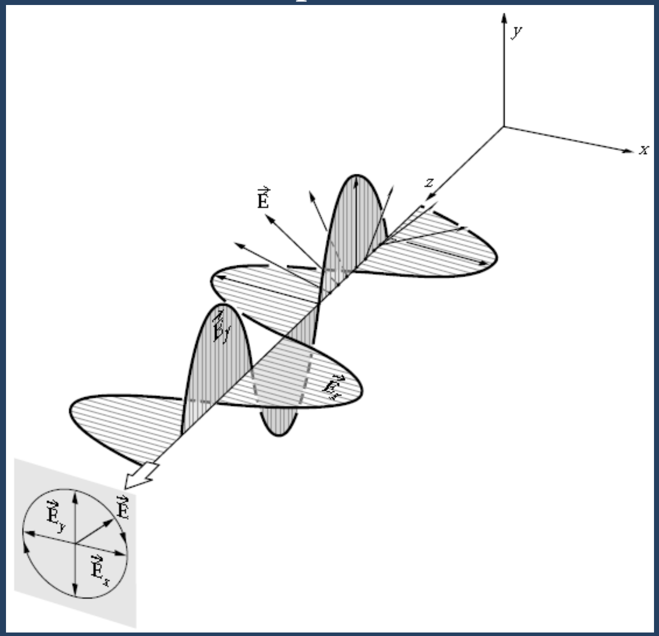
The Polarization of Light

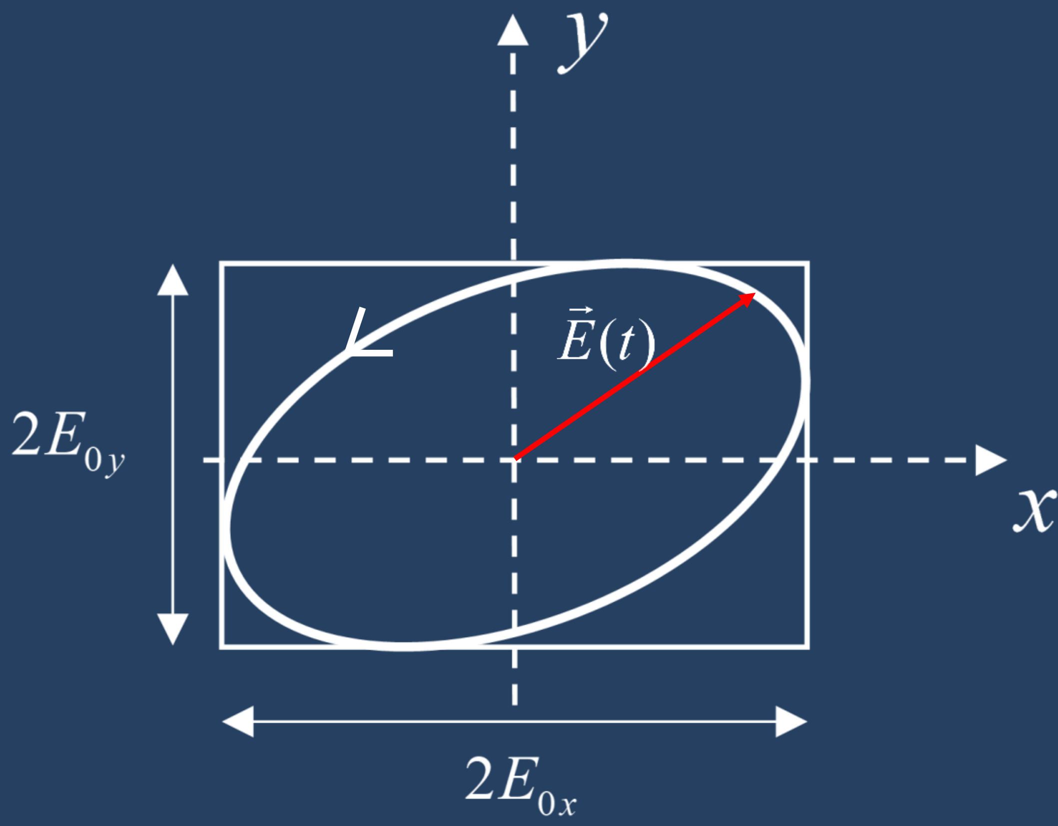
$$\vec{E} = E_{0x}\cos\left(kz - \omega t + \varphi_x\right)\hat{i} + E_{0y}\cos\left(kz - \omega t + \varphi_y\right)\hat{j}$$

Linear polarization



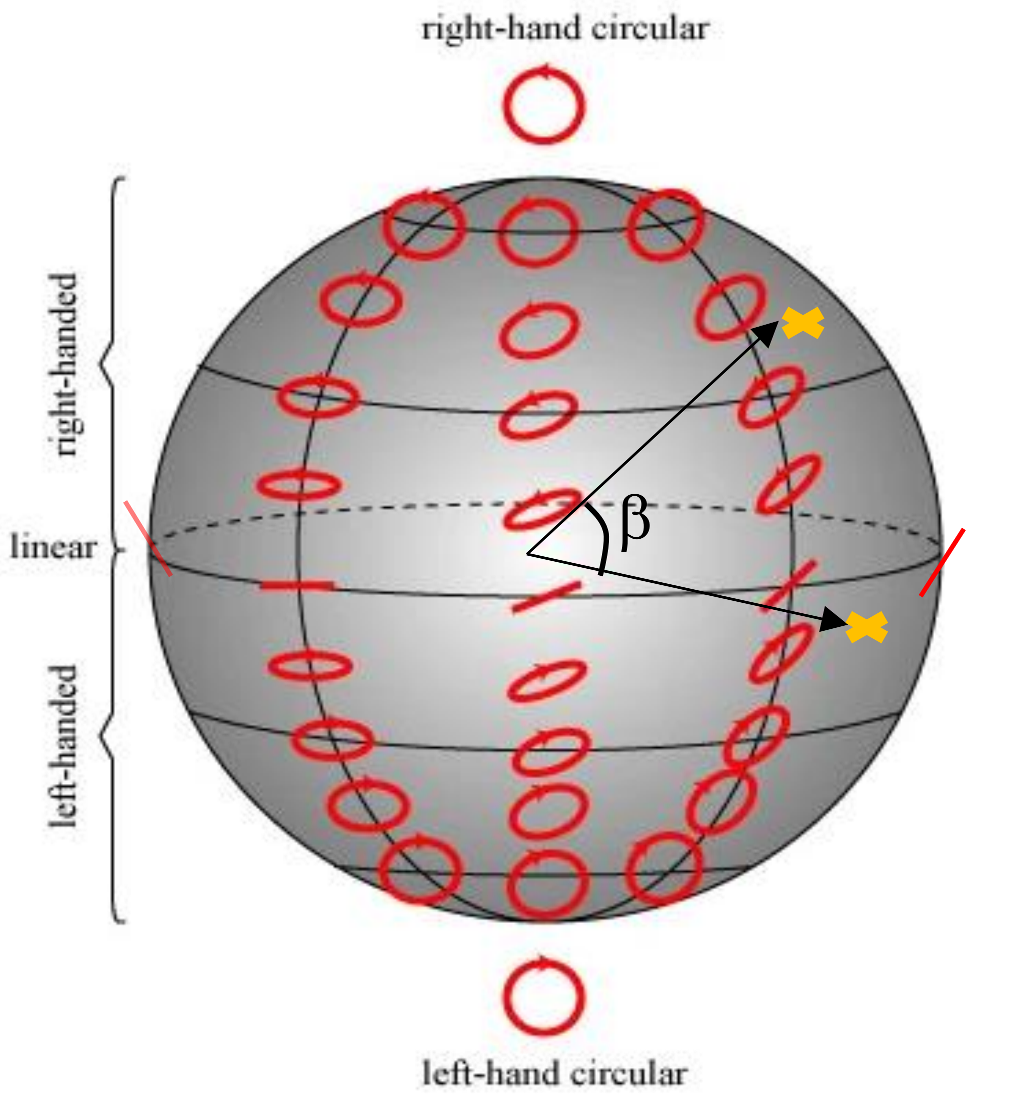
Circular polarization





Ref. Optics, Hecht

The Poincaré sphere



Ref. : https://www.fiberoptics4sale.com/blogs/wave-optics/104501958-poincare-sphere

- Latitude: same ratio between axes
- Longitude: same orientation
- Diametrically opposed states are mutually orthogonal.
- Coherent superposition of states with different polarization can interfere if not mutually orthogonal :

$$I \propto \left\langle E^2 \right\rangle_t$$

$$I = I_1 + I_2 + 2\gamma\sqrt{I_1 I_2}\cos\left(\Delta\varphi\right)$$

$$\gamma = \cos(\beta / 2)$$

Representing Polarization with Jones Vectors

$$\vec{E} = E_{0x}\cos\left(kz - \omega t + \varphi_x\right)\hat{i} + E_{0y}\cos\left(kz - \omega t + \varphi_y\right)\hat{j}$$

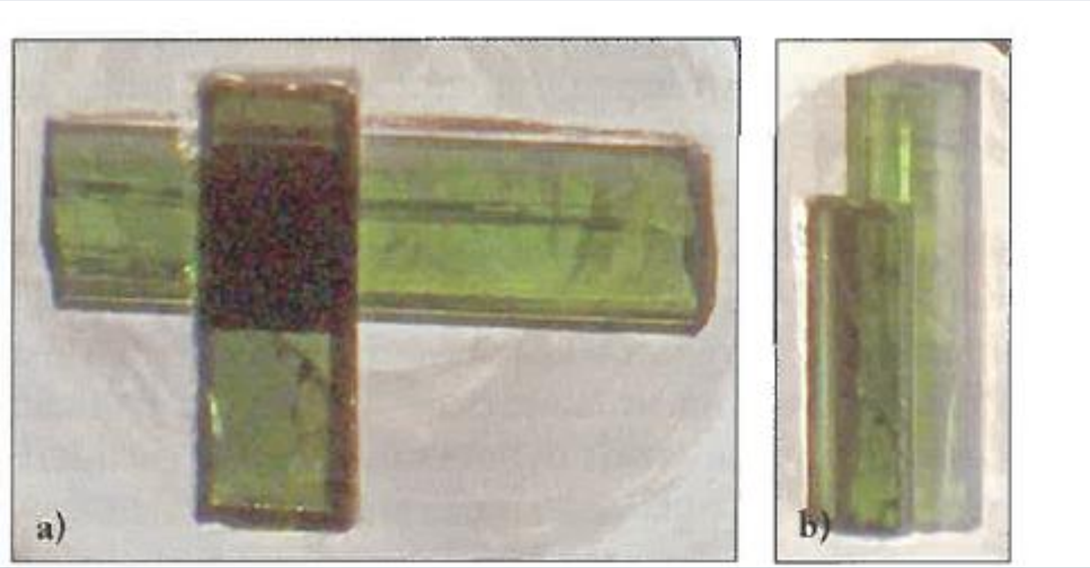
Jones vector: $\vec{u} = E_0 e^{i\varphi_x}\begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)\exp(i\varphi)\end{pmatrix}_{\{H,V\}}$

$$E_0 = \sqrt{{E_{0x}}^2 + {E_{0y}}^2} \qquad \tan(\theta/2) = \frac{E_{0y}}{E_{0x}} \qquad \varphi = \varphi_y - \varphi_x$$

$$\vec{E} = \Re\left\{\vec{u}\exp\left(i\left(kz - \omega t\right)\right)\right\}$$

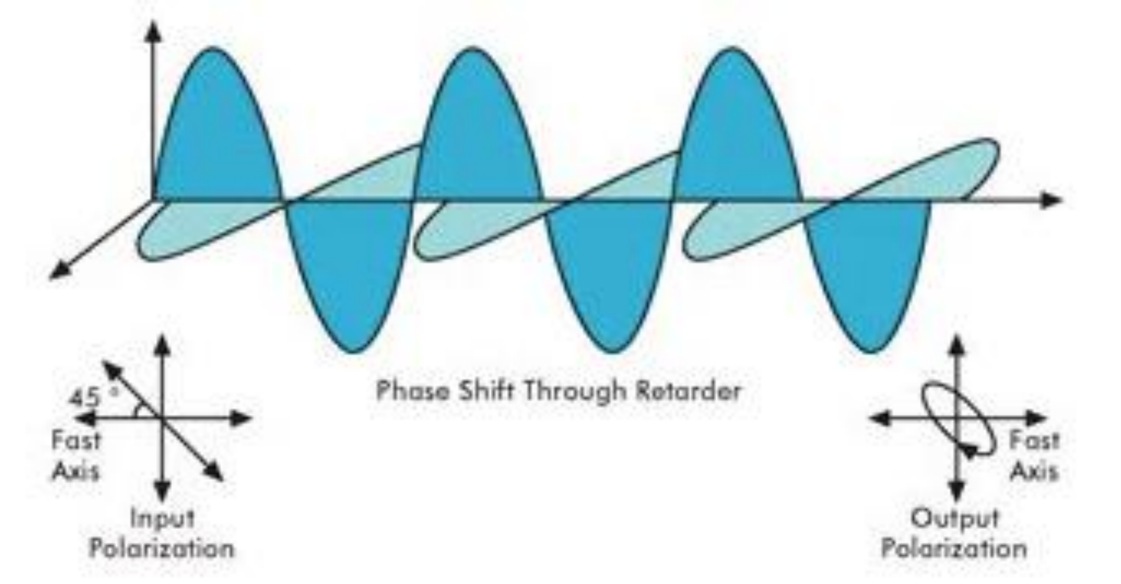
The Manipulation of the Polarization

- Polarizers:



Diattenuation

- Waveplates:



Ref. : https://www.photonics.com/Articles/A_Wave_Plate_for_Every_Application/a38685

Birefringence

Optical elements can be described with matrices

- Homogeneous : orthogonal eigenstates
  - – Diattenuation → **Hermitian** $H = H^{\dagger}$ $H \sim \begin{pmatrix} \exp(k) & 0 \\ 0 & \exp(-k) \end{pmatrix}$
  - – Birefringence → **Unitary** $U^{-1} = U^{\dagger}$ $U \sim \begin{pmatrix} \exp(i\delta) & 0 \\ 0 & \exp(-i\delta) \end{pmatrix}$
- Inhomogeneous: non-orthogonal eigenstates
- Degenerate : only one eigenstate $\mathrm{Tr}\left(J_{\text{degen.}}\right)^2 - 4\mathrm{Det}\left(J_{\text{degen.}}\right) = 0$ $J_{\text{degen.}} = \begin{pmatrix} \lambda & \eta \\ 0 & \lambda \end{pmatrix}_{\{\vec{v}_{EP}, \vec{v}_{\perp}\}}$

Ref.: S.-Y. Lu and R. A. Chipman, *Homogeneous and inhomogeneous Jones matrices*, JOSA A, **11(2)**, 766-773, 1994

Polar Decomposition Theorem

One can always write an arbitrary Jones Matrix as a product of a Hermitian matrix with a unitary matrix.

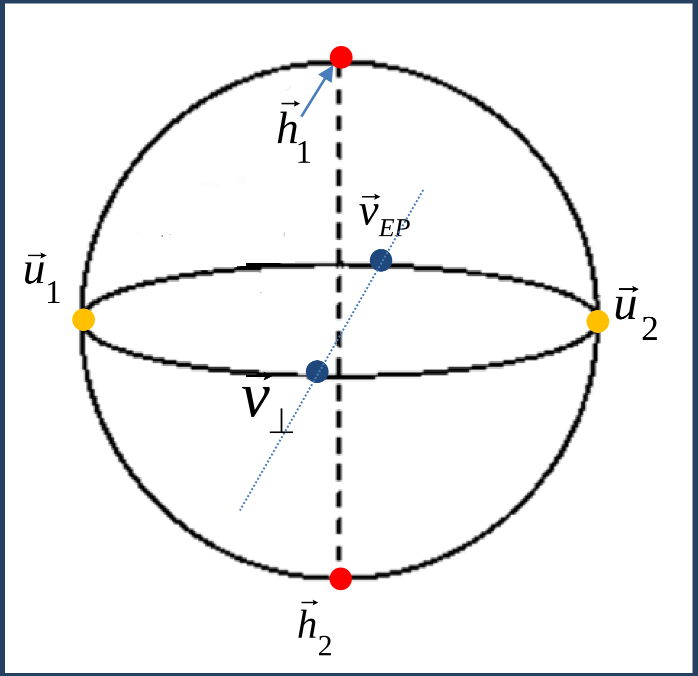
$$J = HU = UH' \quad \text{where} \quad \left[\begin{array}{l} H^2 = J^\dagger J \\ H'^2 = JJ^\dagger \end{array} \right. \quad U = JH^{-1} = H'^{-1} J$$

In the case of inhomogeneous matrices, the eigenvectors of U and H are different. The resulting eigenvectors of J are no longer orthogonal in general.

The relations between H and U for J to be degenerate?

Ref.: S.-Y. Lu and R. A. Chipman, Homogeneous and inhomogeneous Jones matrices, JOSA A, **11(2)**, 766-773, 1994

Obtaining degenerate element with polarizers and waveplates



$$J = HU = UH'$$

$$H \sim \begin{pmatrix} \exp(k) & 0 \\ 0 & \exp(-k) \end{pmatrix}_{\{\vec{h}_1, \vec{h}_2\}}$$

$$U \sim \begin{pmatrix} \exp(i\delta) & 0 \\ 0 & \exp(-i\delta) \end{pmatrix}_{\{\vec{u}_1, \vec{u}_2\}}$$

$$\cos(\delta)\cosh(k) = \pm 1 \quad \text{[eq. (1)]}$$

In order to obtain a degenerate Jones matrix, two conditions must be met:

1. The eigenstates of U are maximally distant from those of H.

$$J_{\text{degen.}} \sim \begin{pmatrix} \lambda & \eta \\ 0 & \lambda \end{pmatrix}_{\{\vec{v}_{EP}, \vec{v}_\perp\}}$$

2. The birefringence and diattenuation must be carefully balanced [eq. (1)].

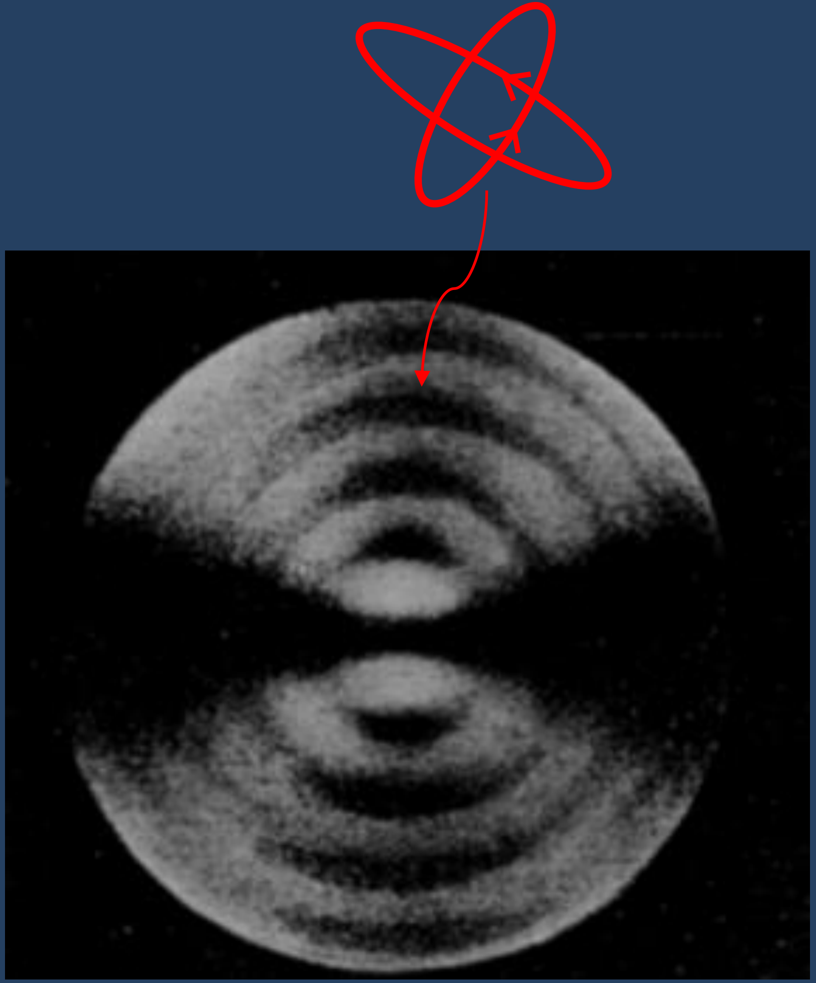
The resulting degenerate eigenstate of J is maximally distant from the other two pairs of eigenstates.

Anomalies in Light Propagation in Anisotropic Crystals

Cordierite

S. Pancharatnam
1934-1969

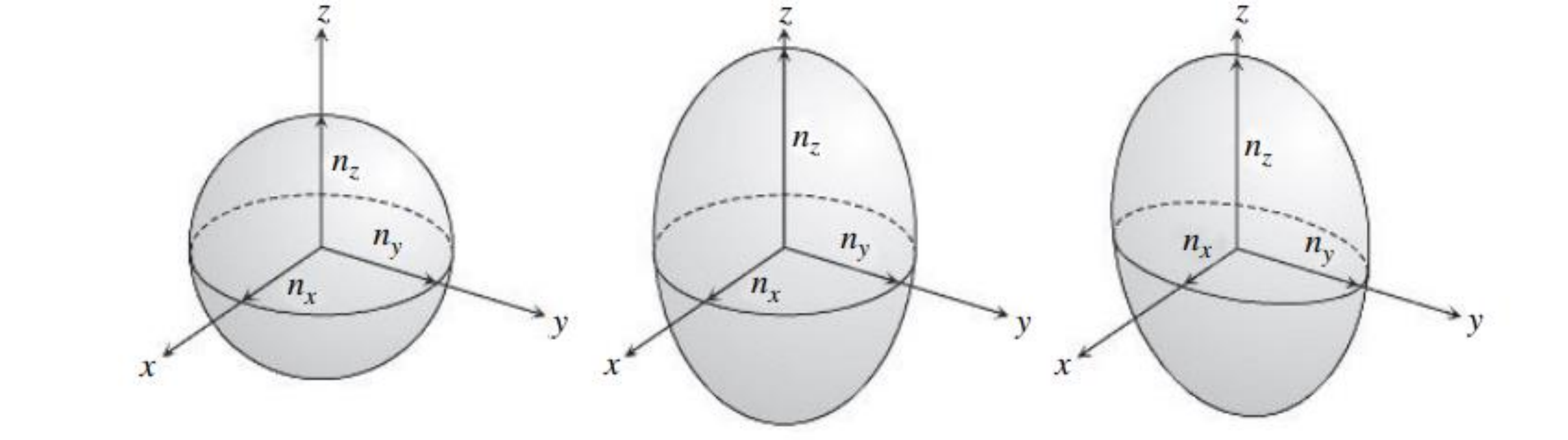
https://geology.com/minerals/cordierite.shtml



- Observation of fringes without analyzer.

Ref.: S. Pancharatnam: The propagation of light in absorbing biaxial crystals II. Experimental, Proc. Indian Acad. Sci. A, 42(5) 235-23 (1955).

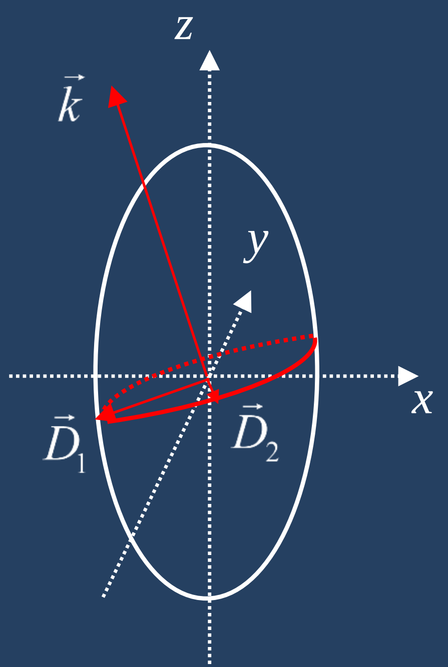
Classification of Optical Materials



(a) isotropic ($n_x = n_y = n_z$) (b) uniaxial ($n_x = n_y < n_z$) (c) biaxial ($n_x < n_y < n_z$)

Ref. Fujiwara, spectroscopic elipsommetry

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

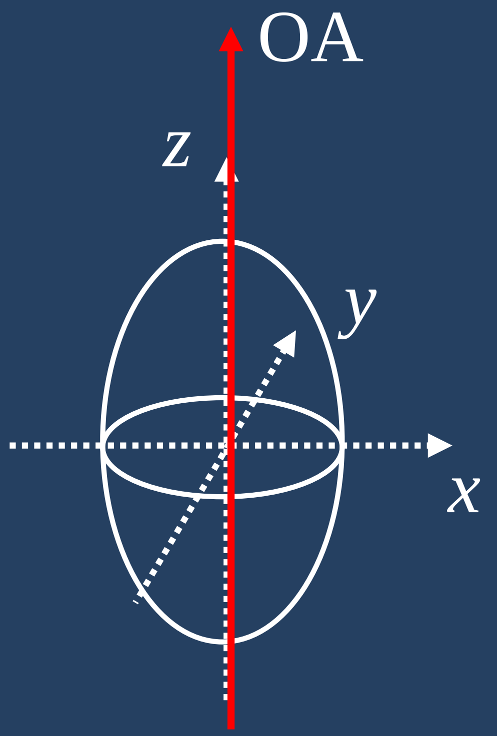


$$n^2 = \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix}$$

$$\vec{D} = \varepsilon_0 n^2 \vec{E}$$

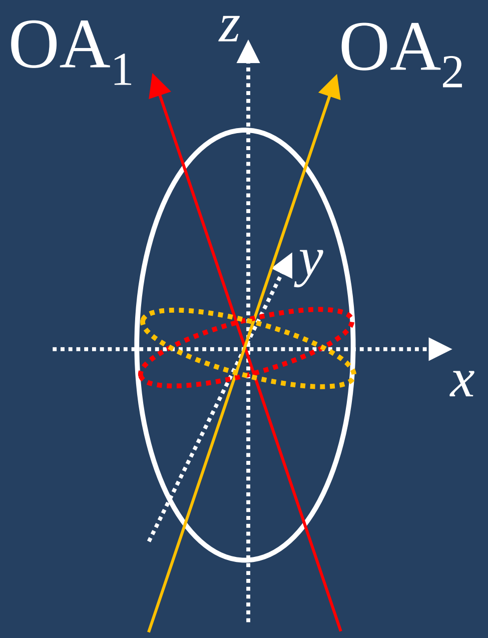
Optical Axes in Anisotropic Crystals

Uniaxial crystal



$$n_x = n_y \neq n_z$$

Biaxial crystal

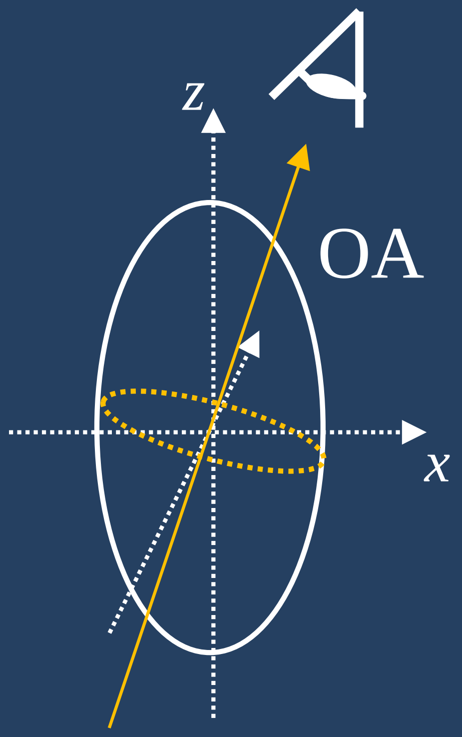


$$n_x < n_y < n_z$$

- In absorbing biaxial materials, there may still exist diattenuation in the direction of the optics axes. So the material remains optically anisotropic even in the direction of the OAs.

Observation of Exceptional Points in the Neighborhood of an Optical Axis

Biaxial crystal



$$n_x < n_y < n_z$$

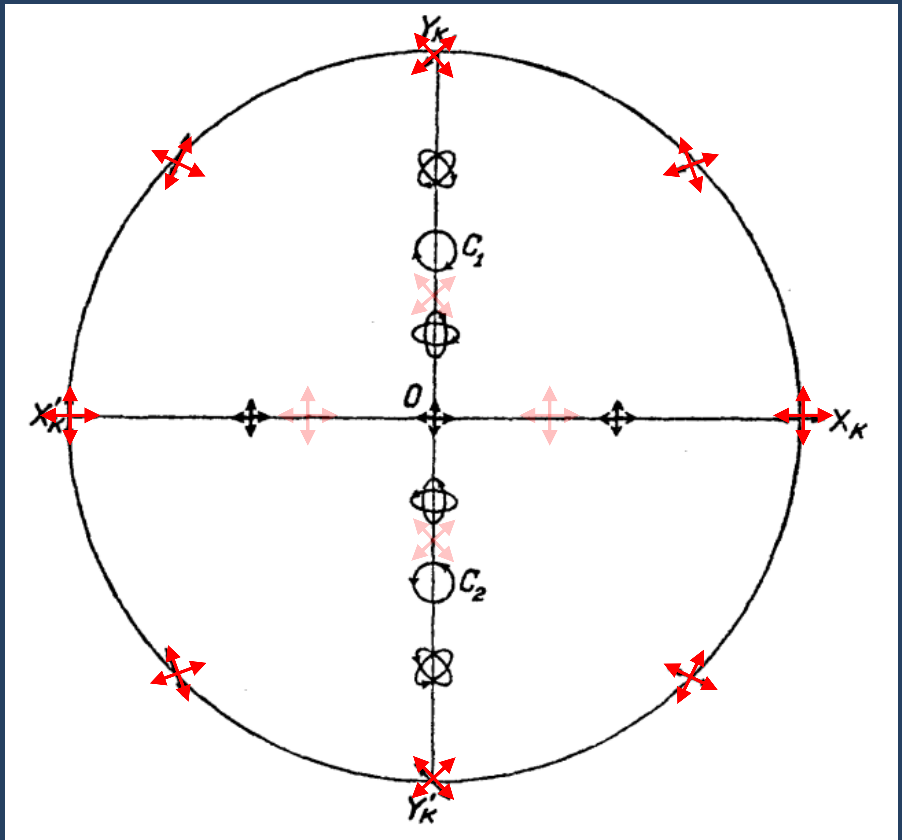
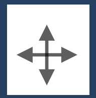


FIG. 1

X_k, X_k' = axial plane; O = optic axis; C_1, C_2 = singular axes.

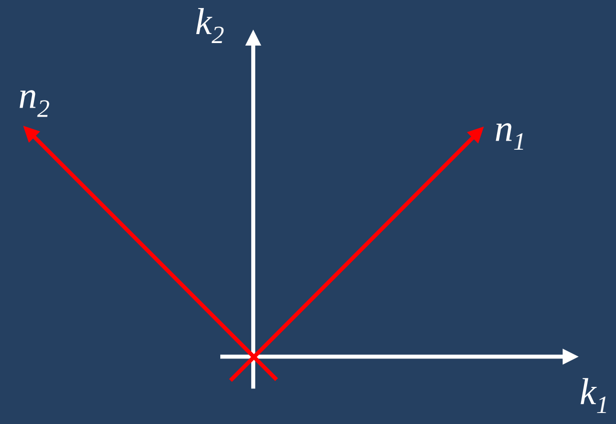
 : eigenstates for birefringence.

 : eigenstates for diattenuation.

 : net eigenstates accounting for birefringence and diattenuation.

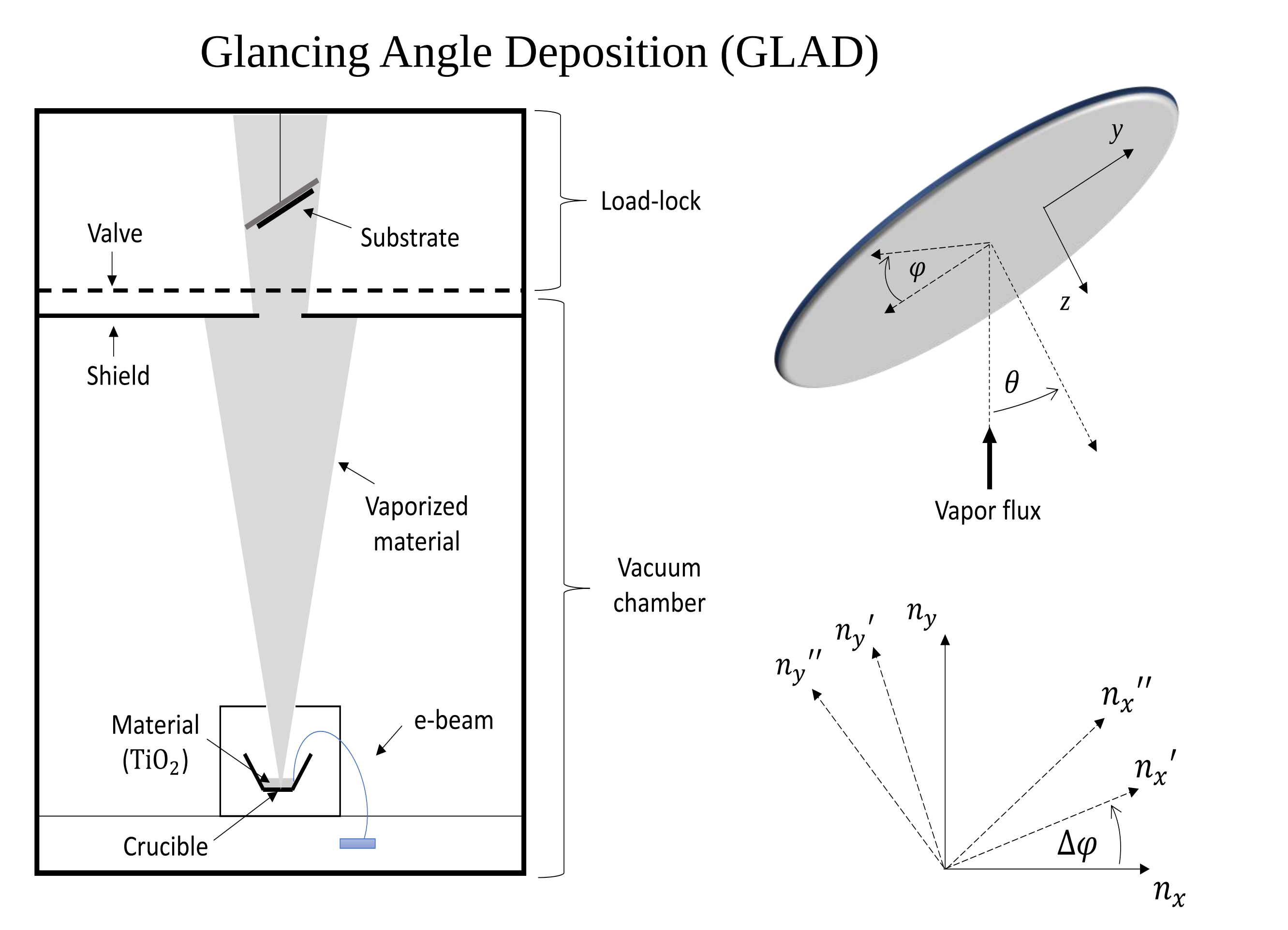
xoz plane

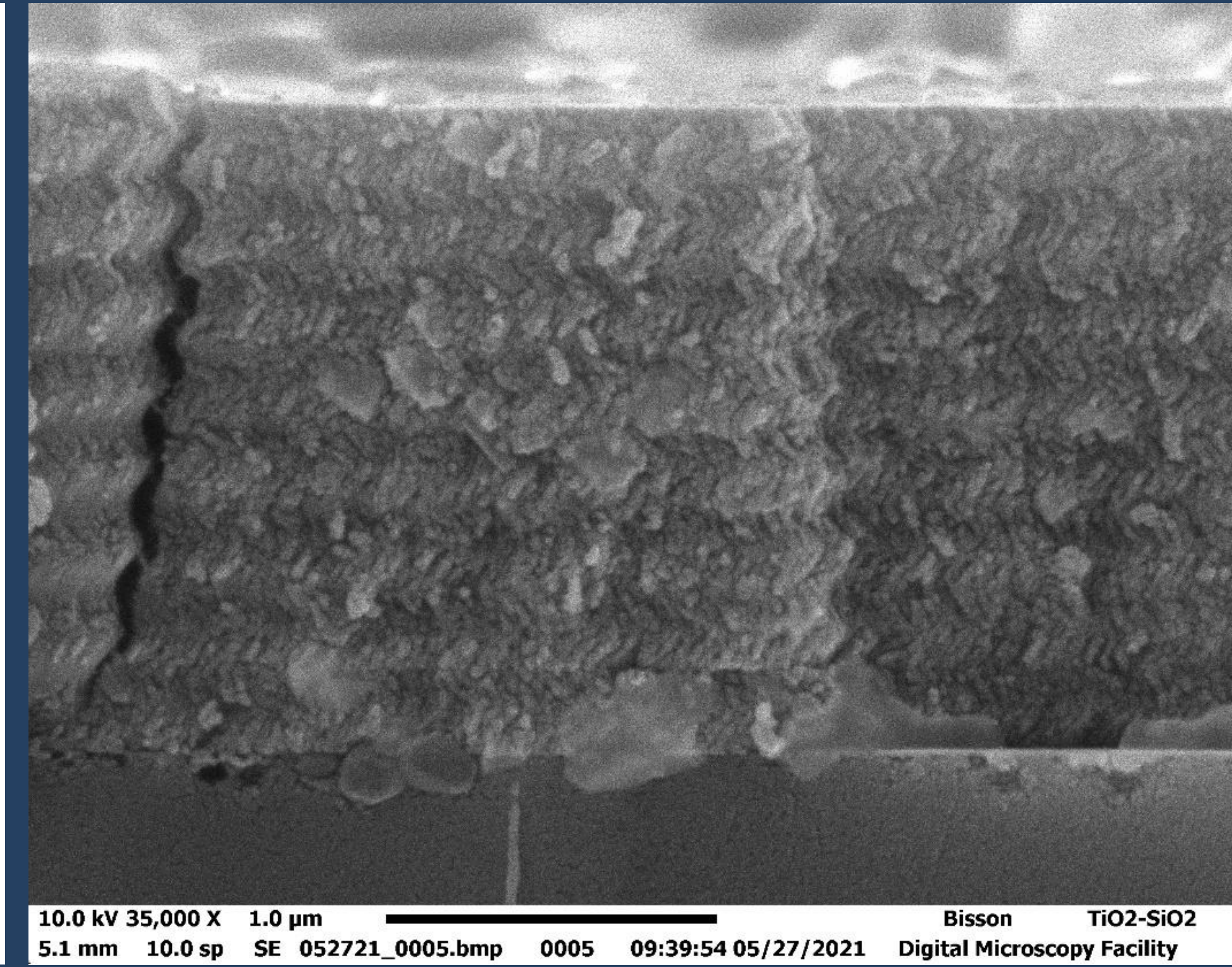
Perpendicular to xoz plane



Ref.: S. Pancharatnam: The propagation of light in absorbing biaxial crystals II. Experimental, Proc. Indian Acad. Sci. A, 42(5) 235-23 (1955).

Helically-Structured Thin Films





Ref.: Gallant et al., Opt. Lett. 46(13) 3135 (2021)

Methodology

- We use ellipsometry to measure the elements of the Jones matrix in reflection.
- We calculate its eigenvalues and eigenvectors.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u_{j1} \\ u_{j2} \end{bmatrix} = \omega_j \begin{bmatrix} u_{j1} \\ u_{j2} \end{bmatrix}$$

$$z_j \equiv \frac{u_{j2}}{u_{j1}}, \quad j \in \{1, 2\}$$

$$\omega_j^2 - (a_{11} + a_{22})\omega_j + (a_{11}a_{22} - a_{12}a_{21}) = 0$$

$$a_{12} z_j^2 + (a_{11} - a_{22}) z_j - a_{21} = 0$$

Zero at an EP

$$\omega_{\pm} = \frac{(a_{11} + a_{22}) \pm \sqrt{\boxed{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}}{2}$$

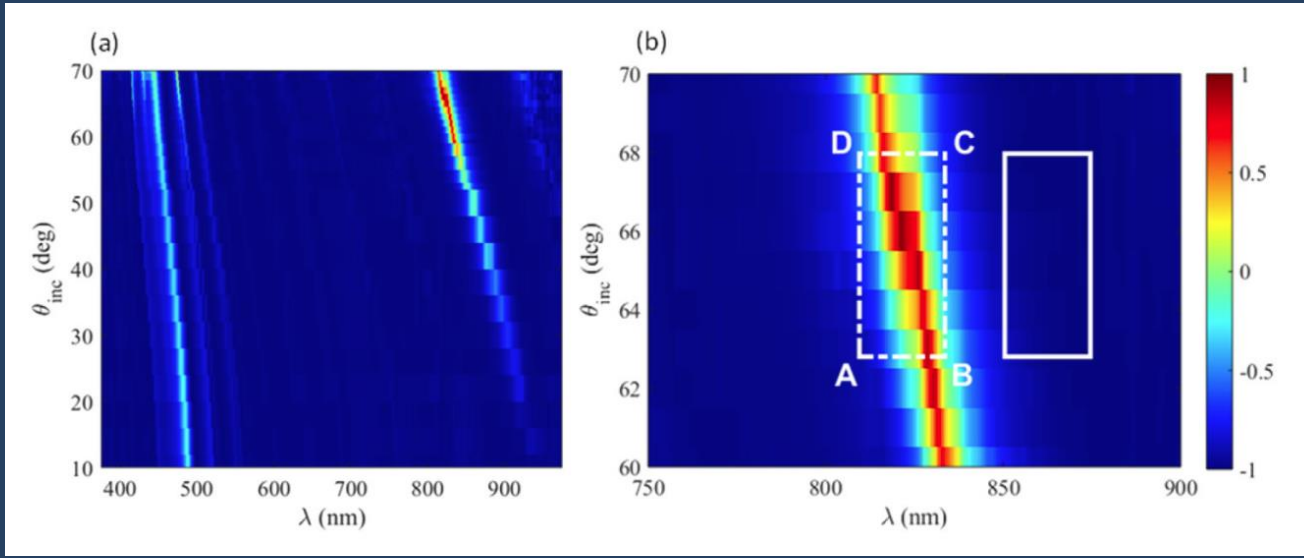
Zero at an EP

$$z_j = \frac{(-a_{11} + a_{22}) \pm \sqrt{\boxed{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}}{2}$$

The conditions for the coalescence of the eigenvectors and eigenvalues are equivalent

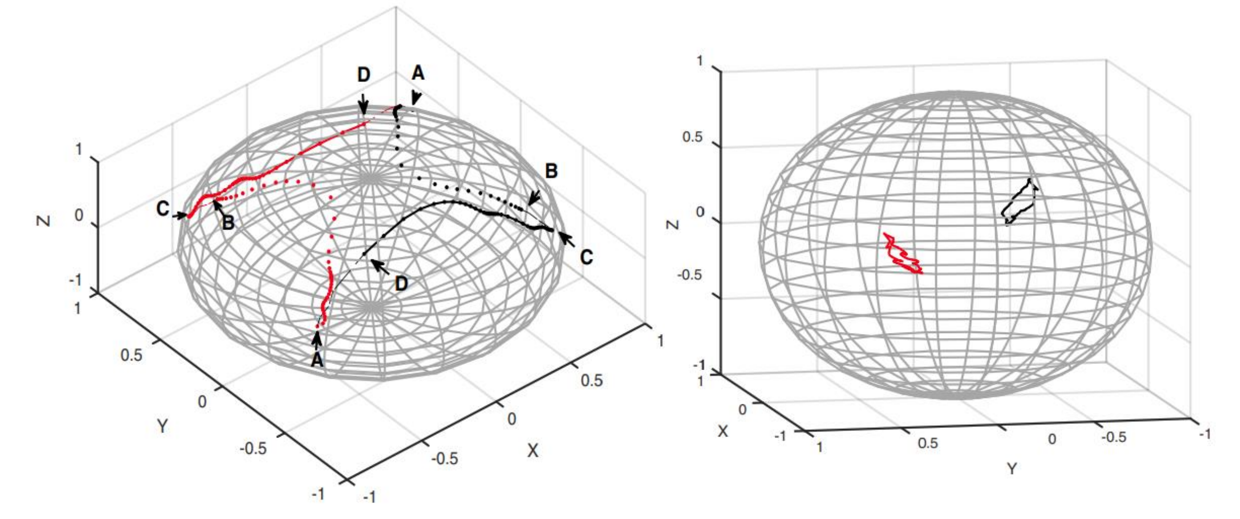
Results

Scalar product of the Stokes eigenvectors



There is a region around λ=800 nm and θ_{inc}=66^o where the proximity of the two eigenvectors on the Poincaré sphere is large.

Parallel transport around an EP



The confirmation of the existence of the an EP is obtained by the switching of the eigenvectors after transport on a loop.

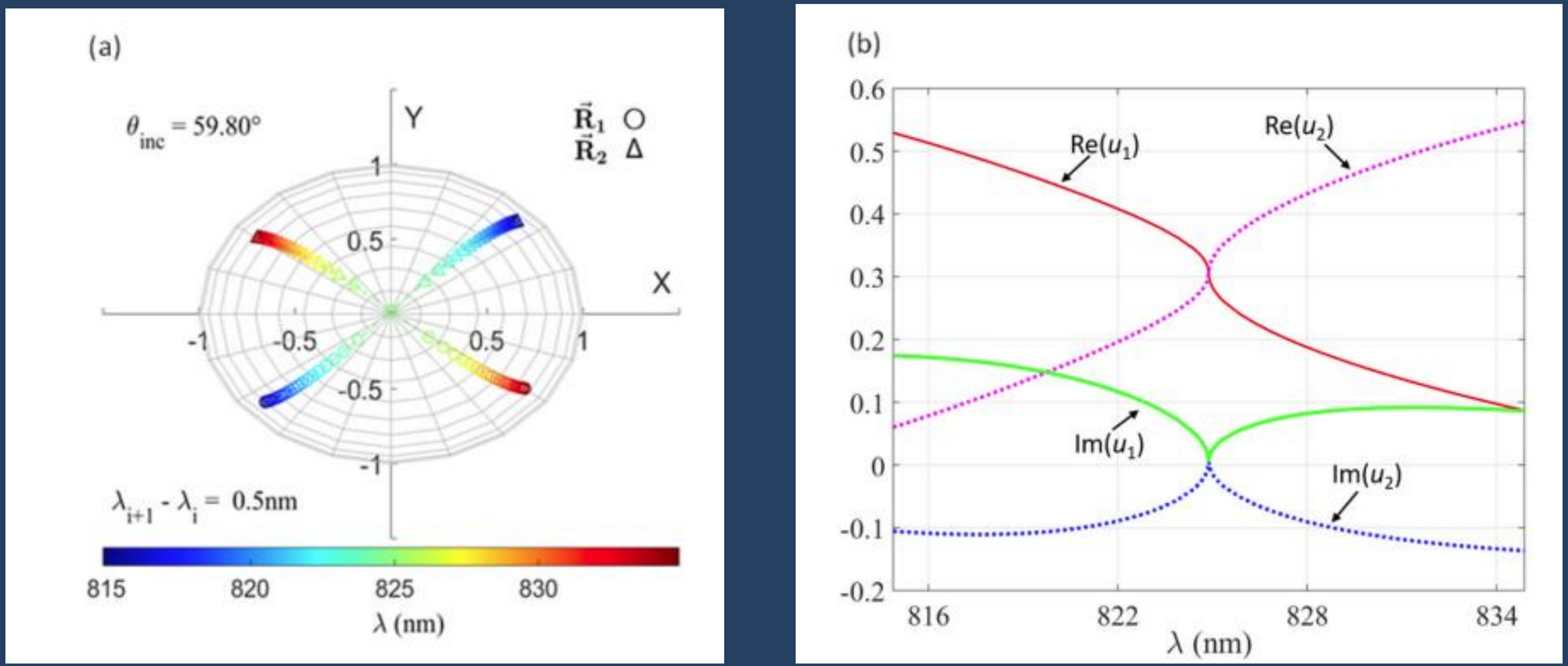
Why is it interesting?

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} u_{j1} \\ u_{j2} \end{bmatrix} = \omega_j \begin{bmatrix} u_{j1} \\ u_{j2} \end{bmatrix}$$

$$\omega_j^{\,2} - \left(a_{11} + a_{22}\right)\omega_j + \left(a_{11}a_{22} - a_{12}a_{21}\right) = 0$$

Zero at a EP

$$\omega_{\pm} = \frac{\left(a_{11} + a_{22}\right) \pm \sqrt{\boxed{\left(a_{11} + a_{22}\right) - 4\left(a_{11}a_{22} - a_{12}a_{21}\right)}}}{2}$$

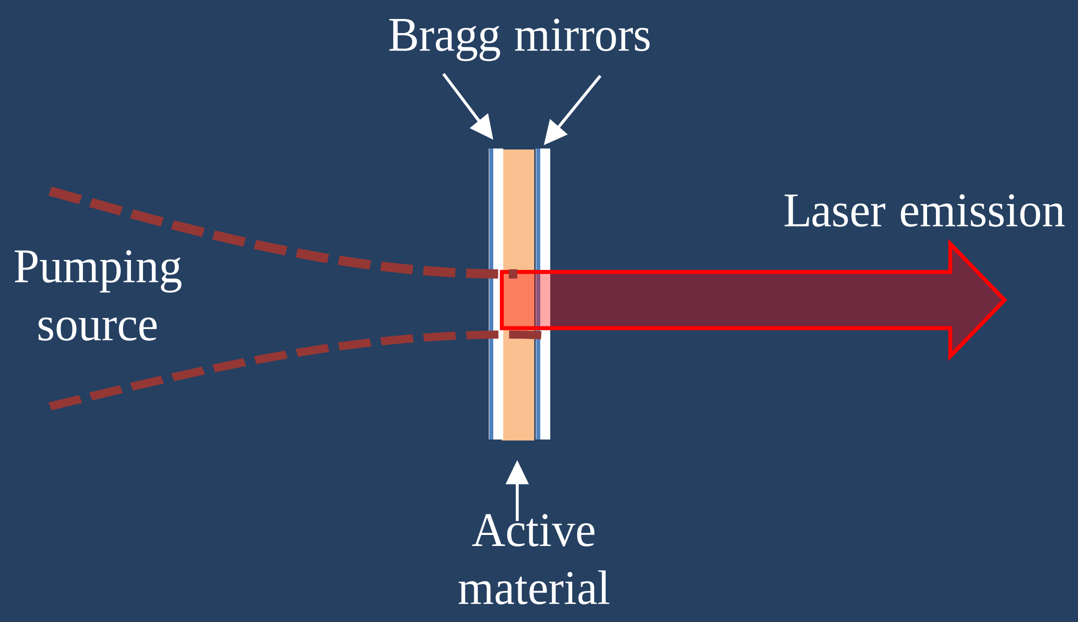


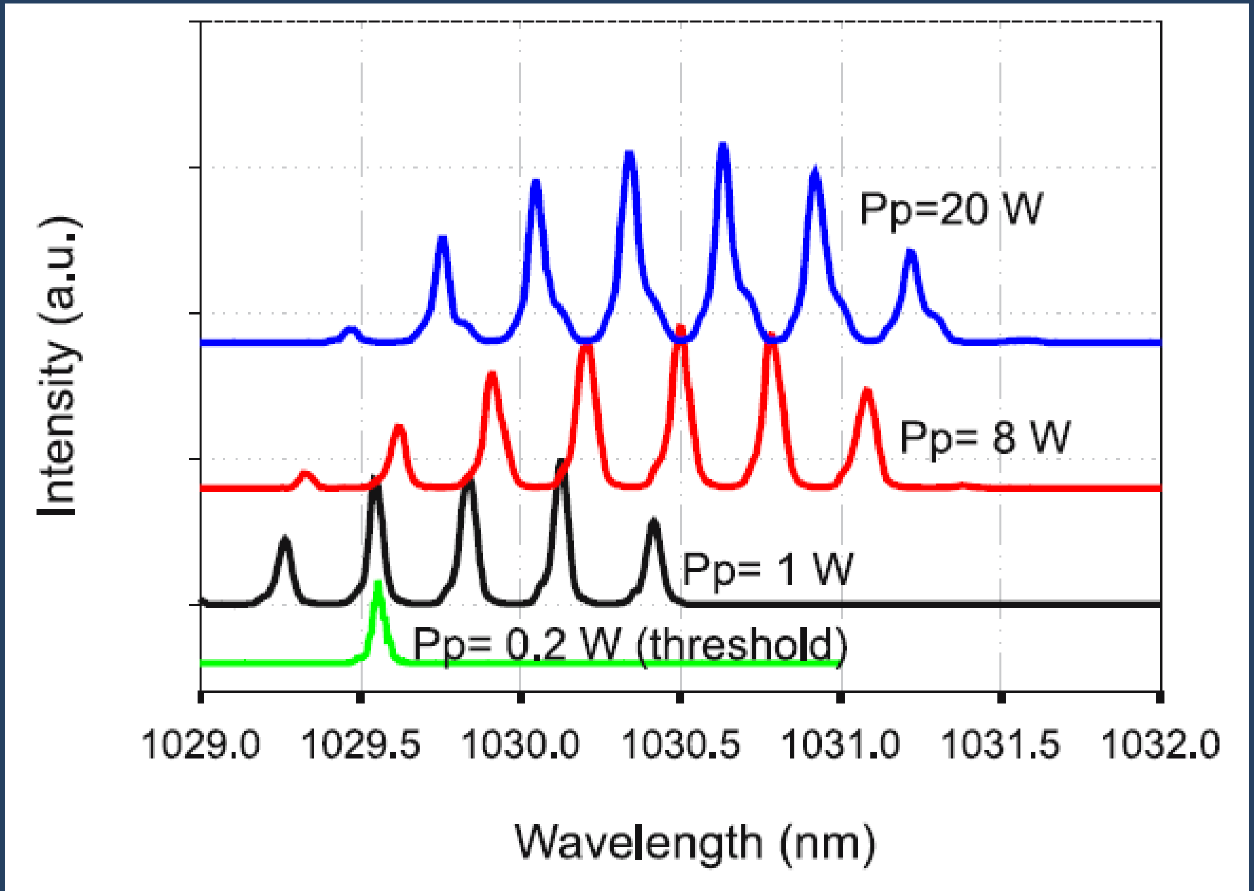
- Enhanced (square-root) sensitivity of the eigenvector/eigenvalue to a change in the Jones matrix in the neighborhood of an EP.
- Experimentally, this requires one to constantly adjust the incident polarization state to an eigenvector of the Jones Matrix as the perturbation takes place.

How do we force light to be an eigenstate of a Jones matrix?

Answer: use active devices

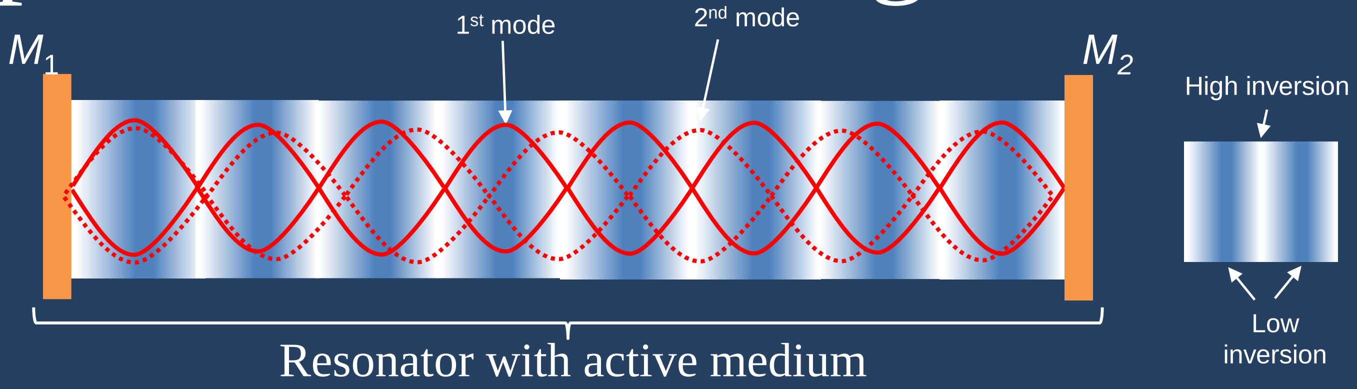
How to achieve single frequency emission with a microchip laser?





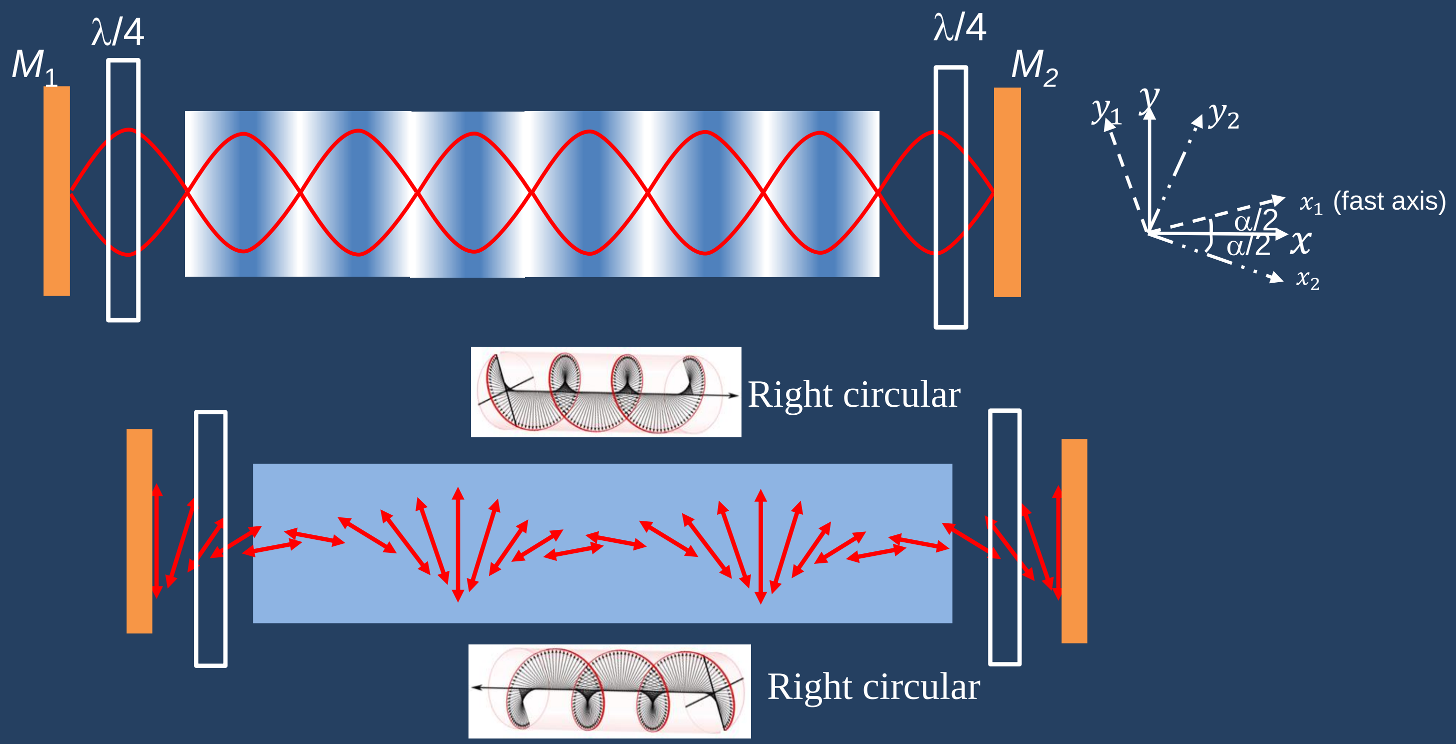
Ref.: Bisson et al., Appl. Phys. B 85(4), 519-524 (2006).

Spatial Hole Burning in Lasers



The twisted mode laser

(Réf.: Siegman et al., Appl. Opt. 1965)



Parity-time (PT) Reflection Symmetry in Quantum Mechanics

- Conventional knowledge: a Hamiltonian must be Hermitian
- A Hamiltonian can have a real eigenvalue spectrum and conserve probability *if it satisfies parity-time reflection symmetry* (Bender and Boettcher PRL **80(24),** 5243, 1998).

$$\hat{P}: \ \hat{x} \to -\hat{x} \qquad \hat{p} \to -\hat{p}$$

$$\hat{T}: \hat{x} \to \hat{x} \qquad \hat{p} \to -\hat{p} \qquad i \to -i$$

$$[\hat{P}\hat{T}, \hat{H}] = 0$$

- $\hat{P}\hat{T}$ is not a linear operator => $\hat{P}\hat{T}$ and $\hat{H}$ need not share the same eigenvectors
  - *Unbroken PT symmetry*
    - All eigenvectors of *H* are simultaneously eigenvectors of *PT* operator
    - Entirely real eigenvalue spectrum
  - Otherwise: *broken* PT *symmetry*

PT-symmetric Jones Matrices

- A 2 x 2 Jones matrix, *J*, is PT-symmetric if it commutes with PT: $(PT)J - J(PT) = 0$

Ex.: $$P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a^* \\ b^* \end{pmatrix}$$

- General form of a PT-symmetric 2 x 2 matrix (Wang, J. Phys. A: Math. Theor. **43**, 295301, 2010):

$$J_{PT} = \begin{pmatrix} A + B\cos\theta - iC\sin\theta & (B\sin\theta + iC\cos\theta + iD)\exp(-i\varphi) \\ (B\sin\theta + iC\cos\theta - iD)\exp(i\varphi) & A - B\cos\theta + iC\sin\theta \end{pmatrix}$$

$$A, B, C, D \in \mathbb{R}$$
$$0 \le \theta < \pi$$
$$0 \le \varphi < 2\pi$$

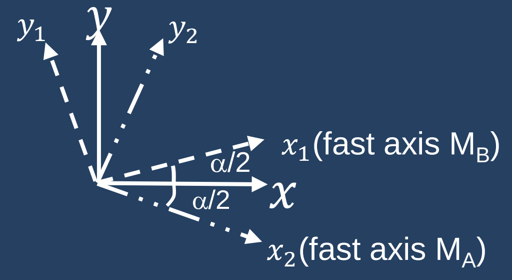
- Control parameter: $\chi \equiv C^2 / (B^2 + D^2)$

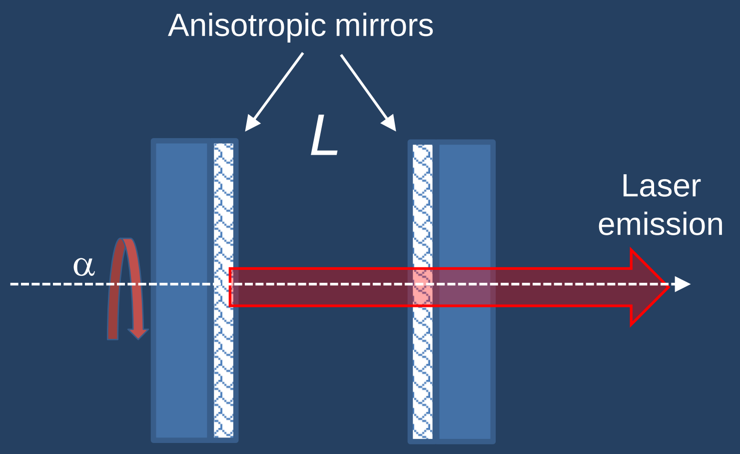
- χ value determines the degree of non-hermiticity of J_{PT}:
  - $\chi \le 1$: unbroken PT-symmetry => eigenvalues are real
  - $\chi > 1$: broken PT-symmetry => eigenvalues are complex conjugate
  - $\chi = 1$: coalescence of eigenstates (exceptional point)

Jones Matrix Analysis of the Polarization Eigenstates

We use a pair or linearly birefringent and diattenuating mirrors:

$$M_1 = \begin{pmatrix} r_{11} & 0 \\ 0 & r_{12} \end{pmatrix}_{x_1 y_1} \qquad M_2 = \begin{pmatrix} r_{21} & 0 \\ 0 & r_{22} \end{pmatrix}_{x_2 y_2}$$





We calculate the round-trip Jones' Matrix:

$$J_{PT} = \begin{pmatrix} A + B\cos\theta - iC\sin\theta & (B\sin\theta + iC\cos\theta + iD)\exp(-i\varphi) \\ (B\sin\theta + iC\cos\theta - iD)\exp(i\varphi) & A - B\cos\theta + iC\sin\theta \end{pmatrix}$$

$$A = \frac{1}{4}\left[(r_{21}+r_{22})(r_{11}+r_{12})\cos(2\alpha) + (r_{21}-r_{22})(r_{11}-r_{12})\right] \qquad B = \frac{r_{21}r_{11} - r_{22}r_{12}}{2}\cos\alpha \qquad C = -\frac{1}{4}\left[(r_{21}+r_{22})(r_{11}+r_{12})\sin(2\alpha)\right] \qquad D = \frac{r_{22}r_{11} - r_{21}r_{12}}{2}\sin\alpha$$

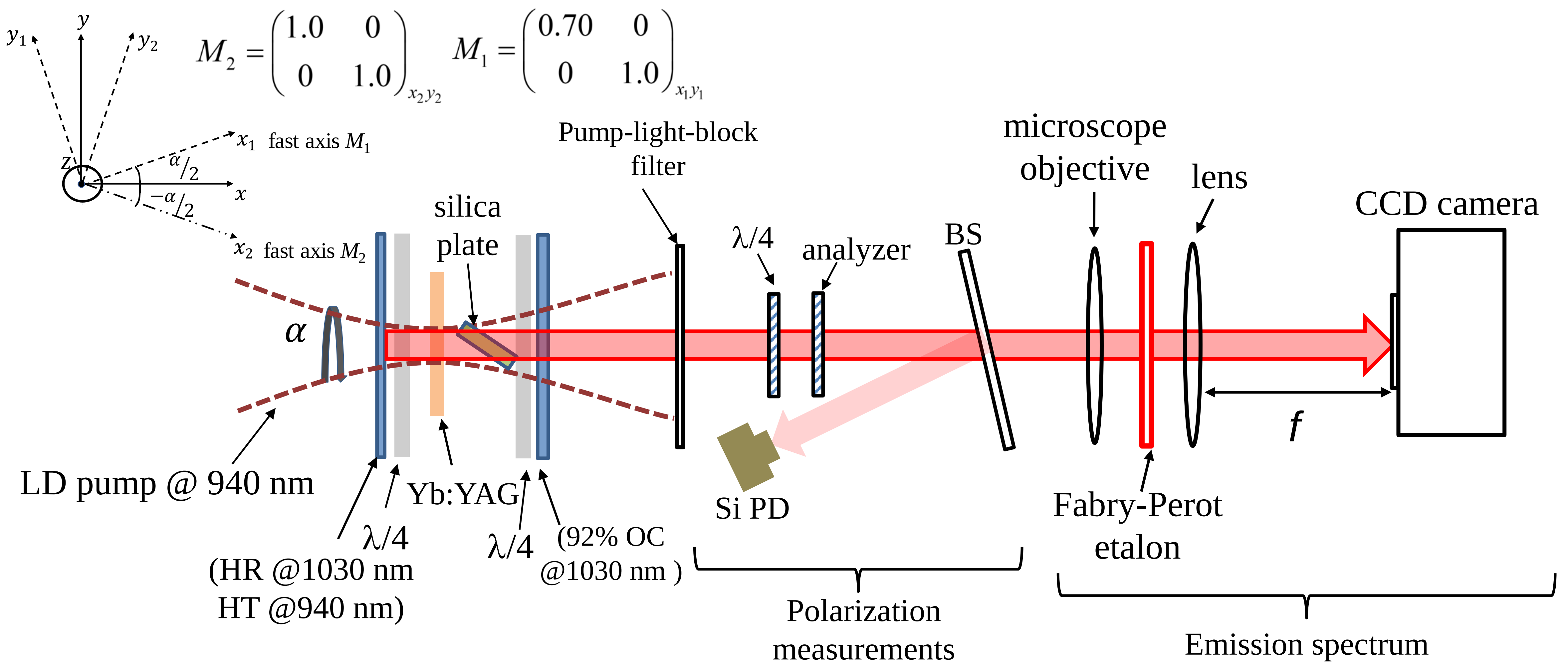
$$\theta = \pi/2 \qquad \varphi = 0$$

- The round-trip Jones matrix is PT-symmetric if …

the r_{ij} have a π phase difference and if diattenuation exists ($r_{11} \neq r_{12}$, …)

- α can be used to control $\chi \equiv C^2 / (B^2 + D^2)$ (exceptional point at χ=1)

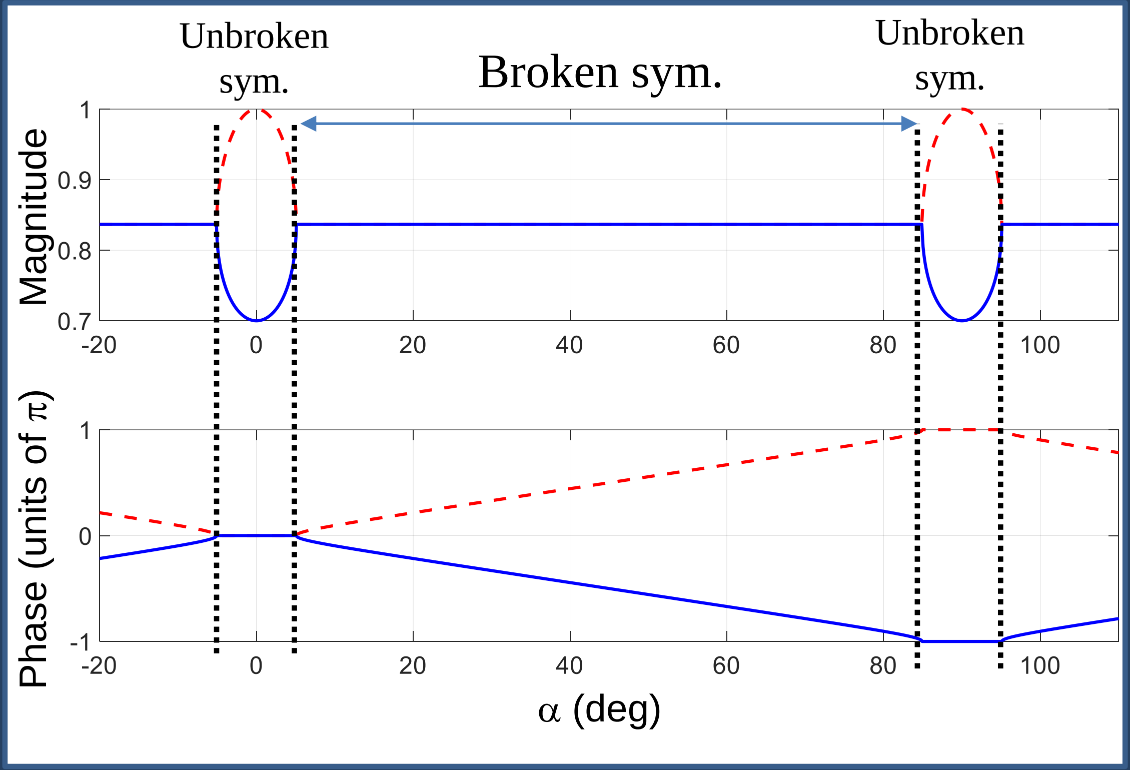
PT-symmetric Laser With Diattenuation



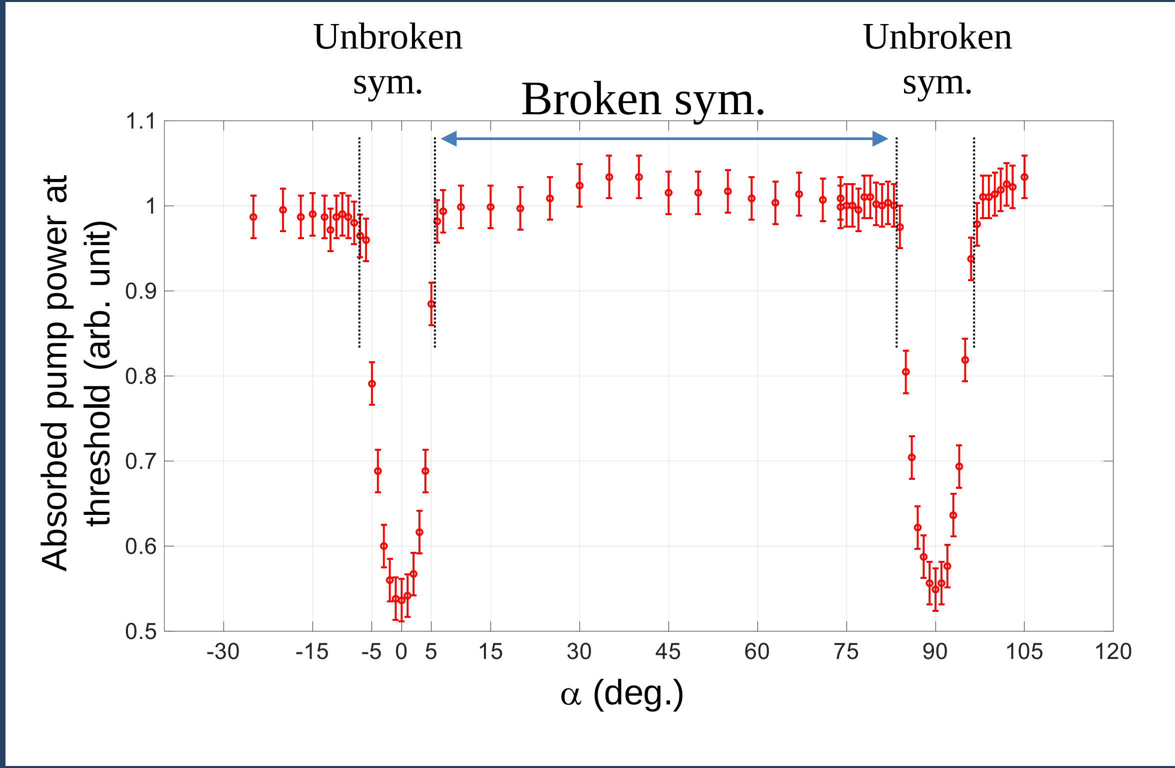
Insert a silica window for diattenuation and $\lambda/4$ waveplates for a π phase shift

Eigenvalues and Threshold of Laser Oscillation

Eigenvalue spectrum



Threshold (exp.)

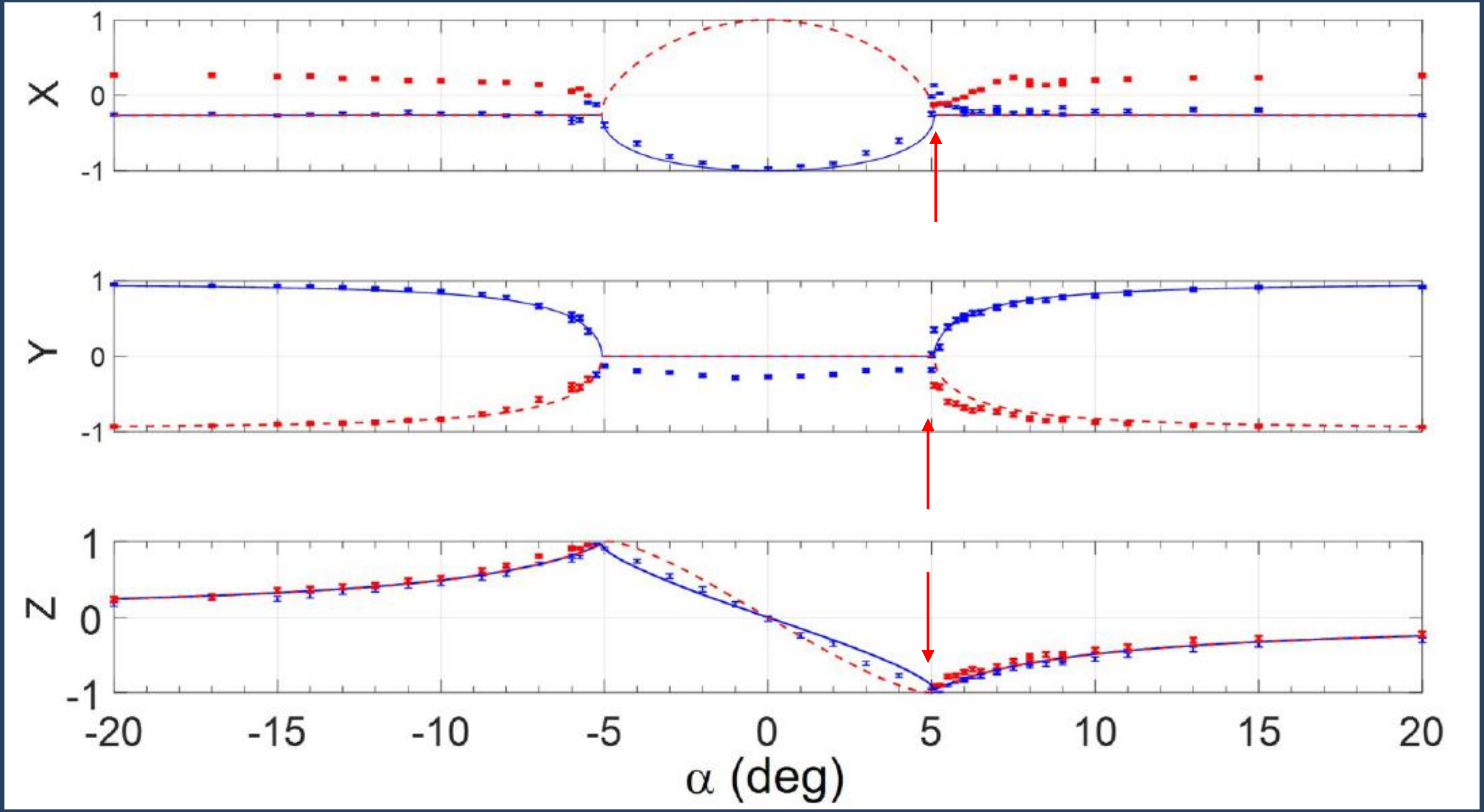


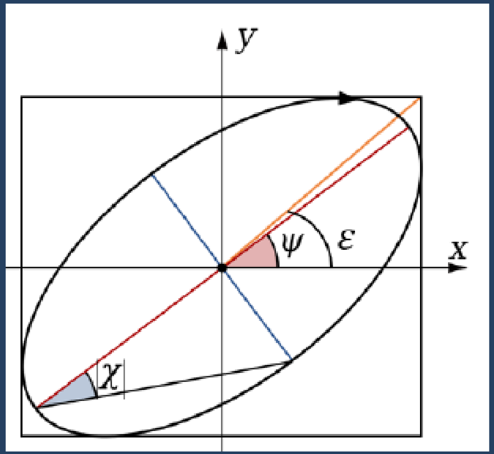
- Sharp drop of the threshold of oscillation in the unbroken region
- Consistent with the larger magnitude of one eigenvalue.

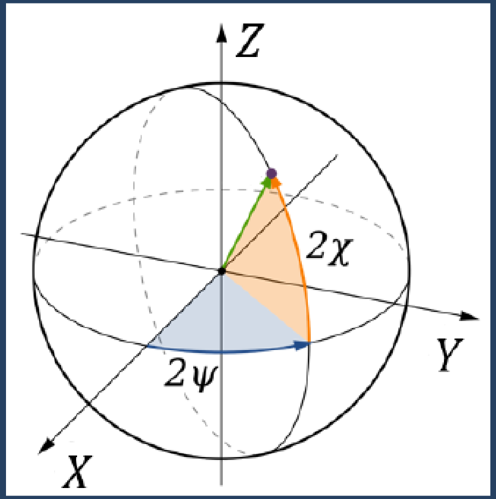
Bisson et al., Phys. Rev. A, 102, 043522, 2020

Coalescence of the Polarization States at the EP

Theory vs experiments

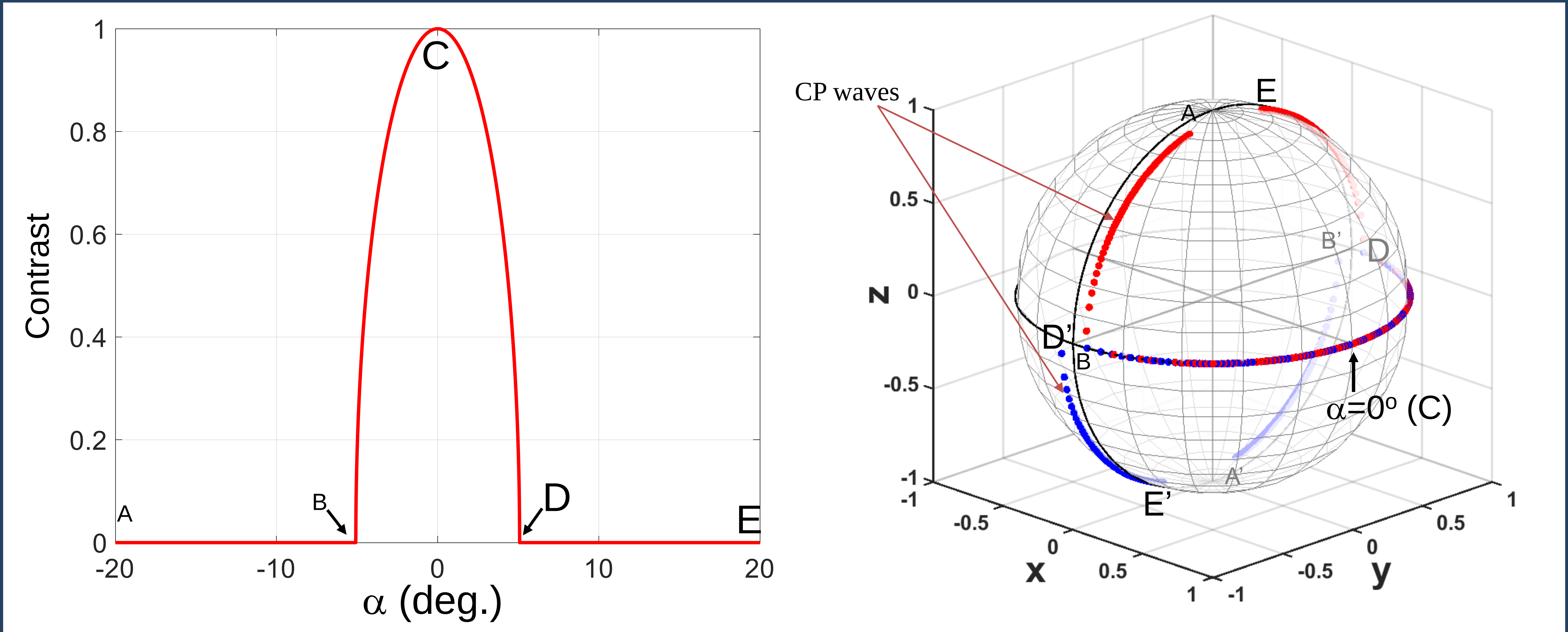






The polarization states are found to merge at about $\alpha_{EP}=\pm 5^o$

Contrast of the Standing Wave



For each mode, the intensity contrast of the standing wave drops to zero in the broken region.

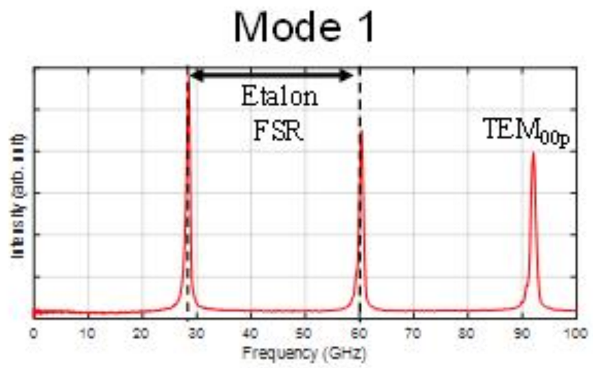
Spatial hole burning is suppressed in the broken PT-symmetry region.

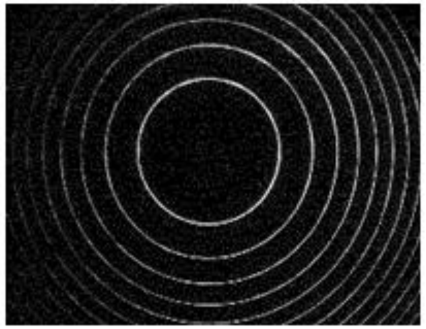
Emission Spectra

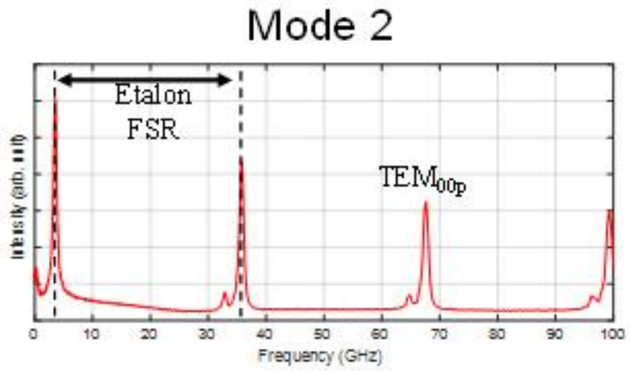
Broken PT symmetry region

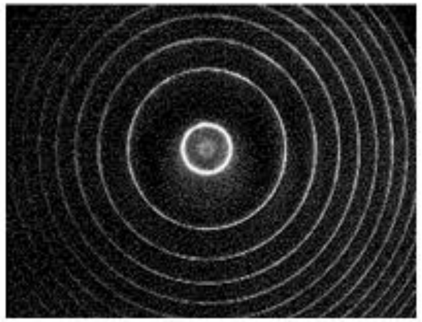
Unbroken PT symmetry region

α=15^{o}

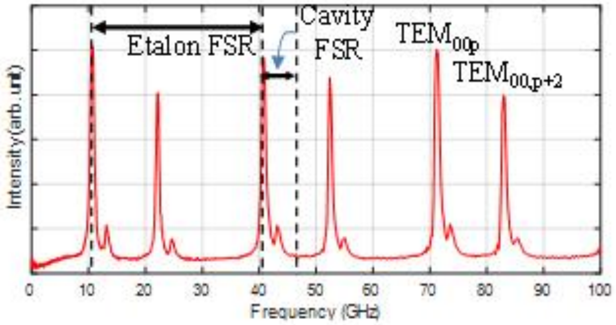


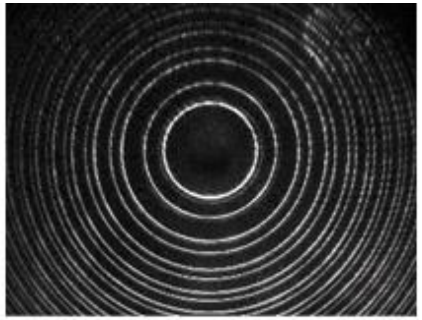




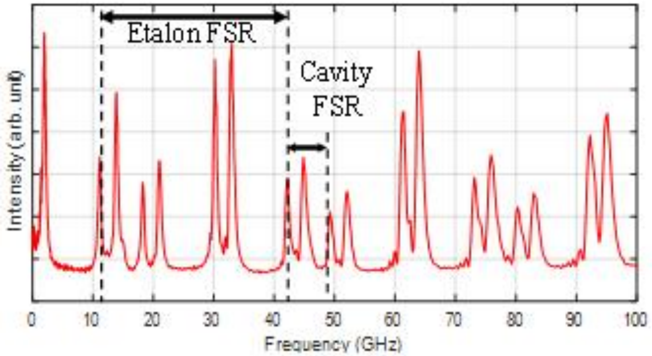


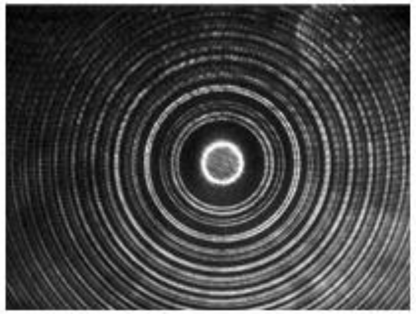
α=5^{o}





α=0

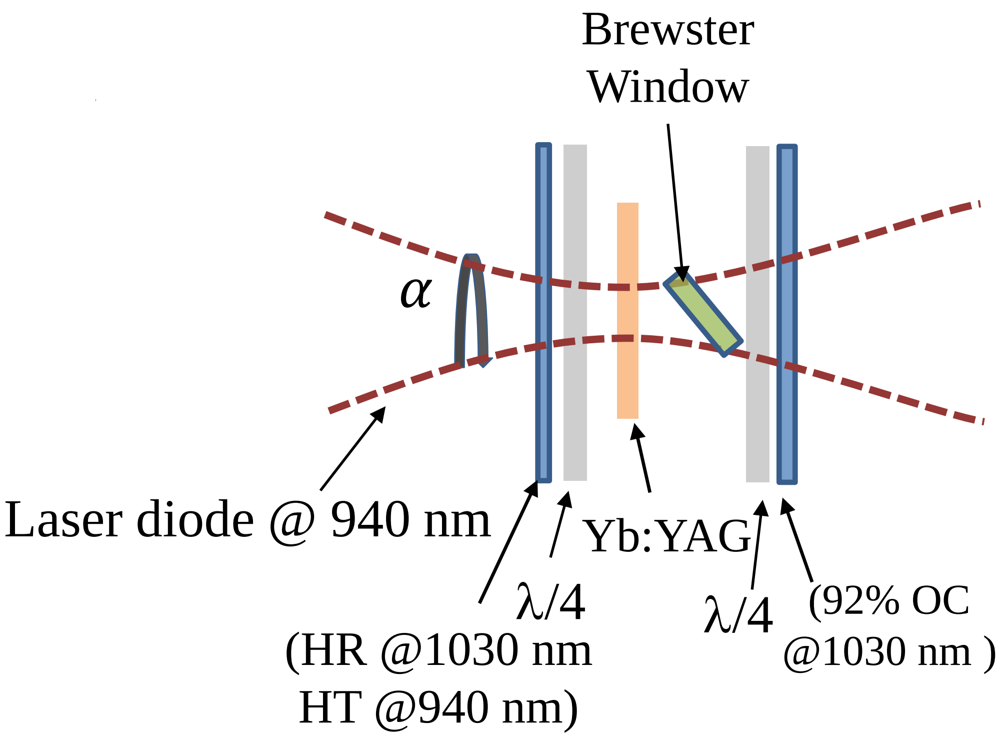


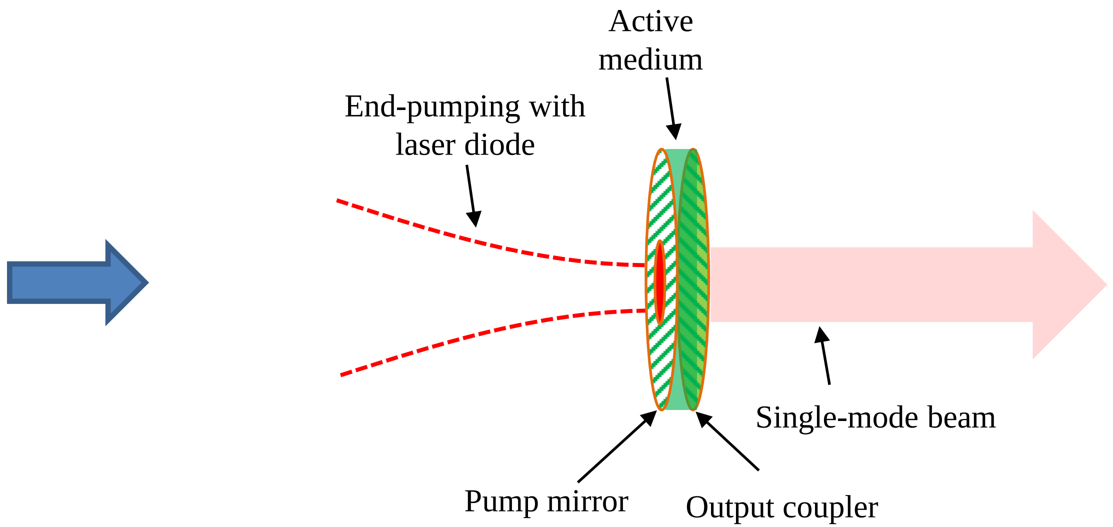


For $\alpha > \alpha_{PE}$: single mode emission, two polarization states

For $\alpha < \alpha_{PE}$: multimode emission, one polarization state

The next Step: Miniaturization using nanostructured thin films





Methods:

1. GLAD-made anisotropic thin films.
2. Photolithography on a conventional Bragg mirror

Summary

Anisotropic laser mirrors enable one to achieve a PT-symmetric Jones matrix of the polarization eigenmodes:

	Dual polarization emission	**Multimode emission**
Unbroken PT symmetry $\alpha<\alpha_{EP}$	**Suppressed**	Allowed by SHB
Broken PT symmetry $\alpha>\alpha_{EP}$	Allowed	**Suppressed**
At the EP $\alpha\approx\alpha_{EP}$	?	?

Such device has potential for lasers emitting at a single frequency as well as for optical sensors.

Thank you for your attention!

Acknowledgments

Gabriel Gallant
Shawn Lapointe
Koffi Novignon Amouzou
Kris Bulmer
Alexandre Doucet
Pierre St-Onge